International Journal of Heat and Mass Transfer 92 (2016) 884–892

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Curvilinear melting – A preliminary experimental and numerical study

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ARTICLE INFO

Article history:
Received 2 July 2015
Received in revised form 11 September 2015
Accepted 17 September 2015
Available online 2 October 2015

Keywords:
Close-contact melting
Melting probe
Phase-change processes
Lubrication approximation
Finite element method

ABSTRACT

When exploring glacier ice it is often necessary to take samples or implement sensors at a certain depth underneath the glacier surface. One way of doing this is by using heated melting probes. In their common form these devices experience a straight one-dimensional downwards motion and can be modeled by standard close-contact melting theory. A recently developed melting probe however, the IceMole, achieves maneuverability by simultaneously applying a surface temperature gradient to induce a change in melting direction and controlling the effective contact-force by means of an ice screw to stabilize its change in attitude. A modeling framework for forced curvilinear melting does not exist so far and will be the content of this paper. At first, we will extend the existing theory for quasi-stationary close-contact melting to curved trajectories. We do this by introducing a rotational mode. This additional unknown in the system implies yet the need for another model closure. Within this new framework we will focus on the effect of a variable contact-force as well as different surface temperature profiles. In order to solve for melting velocity and curvature of the melting path we present both an inverse solution strategy for the analytical model, and a more general finite element framework implemented into the open source software package ELMER. Model results are discussed and compared to experimental data conducted in laboratory tests.

1. Introduction

Liquid water within or underneath ice sheets on the icy moons of our Solar System, like Saturnian moon Enceladus, are believed to be potential habitats for extraterrestrial life [15]. Direct exploration and in situ analysis of those regions requires advanced accessing technologies [3]. One possible mission scenario is detailed in [8] and relies on a maneuverable melting probe such as the IceMole (IM). The IM has been developed at FH Aachen University of Applied Sciences since 2008 and has already been applied in several field tests on terrestrial glaciers in the European Alps, on Iceland and in Antarctica [2].

The IM achieves maneuverability by simultaneously applying a surface temperature gradient to induce a change in melting direction and controlling the effective contact-force by means of an ice screw to stabilize this attitude change. This enables the IM to target a specific position within the ice. In order to fully exploit the maneuverability and use it for trajectory planning and obstacle avoidance, the response of the IM dynamics to a change in its system configuration has to be predicted.

Influential investigations addressing this question can be found in the work of Aamot [1]. He provides a practical approach to determine the minimum required power for a particular melting velocity. Considering a straightforward energy balance and taking lateral conductive losses into account, the model is a good tool for a preliminary analysis of the melting motion (see for instance [20]). However, depending on the melting probe's level of geometrical and thermo-physical complexity, the melting velocities predicted by Aamot's model are considerably higher than the velocities observed in experiments and do not provide reliable data.

In that case, a detailed understanding of the physical processes within the microscale melting film is essential to be able to quantify both conduction, and convection within the melting film. The amount of heat transfer and the local fluid flow field are relevant to determine the progression of the phase-change interface, namely the melting velocity. A model for these processes is given by the theory of close-contact melting (CCM).

Generally, CCM can be subdivided into internal CCM and external CCM. Internal CCM regards a situation in which the phase-change material (PCM) moves with respect to the heat source. External refers to the case in which the heat source moves with respect to

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2015.09.046
0017-9310/

Nomenclature

(u, w)	melt velocity components in (x, z) directions
(x, z)	coordinates, see Fig. 1
L	half length of the heat source
T	temperature
U	melting velocity profile
U_0	melting velocity at the heat source's center
Re	Reynolds number
Ste	Stefan number
c_p	heat capacity
h	radius of the circumscribed circle of a triangular element
h_m	latent heat of melting
h_m^*	reduced latent heat of melting
k	thermal conductivity
n_z	z-component of the unit normal at the phase interface
p	pressure
q''	heat flux
r	curve radius

Greek symbols

α	thermal diffusivity
δ	melt film thickness
λ	relaxation factor
μ	dynamic viscosity
ϕ	r or U_0
ρ	density

Subscripts

0	at $x = 0$
e	at the exit of the melt film, i.e. $x = \pm L$
i	iteration index
L	liquid state
m	melting point of PCM
S	solid state
w	heat source surface

the PCM. Internal CCM can be found in the field of thermal energy storage systems [21], whereas external CCM, which is also more relevant to our case, can be found in a variety of fields, e.g. geology [11], nuclear technology [7], welding [6], manufacturing technologies [12] and thermal drilling of rocks [5] and glaciers [19].

Previous investigations in the field of external CCM usually assume a straight and downwards oriented (rectilinear) melting motion. The work of [9] considers the effects of an asymmetrically distributed force on the PCM. Although his study addresses internal CCM, its fundamental idea can be transferred to external CCM and provides the baseline thought to extend the original CCM theory to curvilinear melting.

In this paper, the mathematical model for curvilinear melting is derived and discussed. In particular, the effects of a variable contact-force as well as different surface temperature profiles are analyzed. The model can be used to predict both melting velocity, and curvature of the melting path for a specific heat source configuration. An inverse solution strategy for the analytical model and a more general finite element framework implemented into the open source software package ELMER are likewise introduced. Results are discussed and compared to experimental data conducted in laboratory tests.

2. Analysis

2.1. Physical model

The physical situation for a planar heat source placed on a block of phase-change material (PCM) is sketched in Fig. 1. In the context of melting probes in glacial exploration we think of the heat source as a copper melting head and the PCM as glacier ice. We furthermore assume a force F to act on the heat source. This could be given by the weight of the heat source alone, or by an additional force acting on the heat source, or, like in the IM case, by a superposition of both.

When changing the configuration of the heat source, the melting process will almost instantaneously equilibrate into a new quasi-stationary melting state [13]. We will hence assume a time-independent temperature profile $T_w(x)$ along the surface of the heat source. If the surface temperature exceeds the melting temperature of the PCM, the PCM in the vicinity of the probe will melt and the heat source will eventually descent into the PCM. As a result, a portion of the produced melt will be squeezed out at both sides of the heat source because the ambient pressure p_e is assumed to be lower than the additional pressure in the fluid field induced by the contact-force. Only a thin melt film of thickness $\delta(x)$ remains between the heat source and the phase interface separating liquid and solid part of the PCM.

2.2. Mathematical model

Since the thickness of the melt film is assumed to be very small (i.e. $\delta/L \ll 1$), we can use lubrication approximation to simplify the governing equations. Scale analysis shows that inertia terms are negligible compared to the pressure gradient and also $\partial/\partial x^2 \ll \partial/\partial z^2$ as long as δ/L and $(\delta/L)\mathrm{Re} \ll 1$. Details regarding the scale analysis can be found in [14] and [18]. Incorporating the stated assumptions simplifies the balance laws for mass, momentum and energy:

$$\textit{Continuity}: \quad \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\textit{Momentum}: \quad \frac{dp}{dx} = \mu_L \frac{\partial^2 u}{\partial z^2}, \quad \frac{dp}{dz} = 0 \tag{2}$$

$$\textit{Energy}: \quad u\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial z} = \alpha_L \frac{\partial^2 T}{\partial z^2} \tag{3}$$

Here α_L is the thermal diffusivity and μ_L is the dynamic viscosity of the melt, hence the liquid phase of the PCM. The components of the local velocity field of the melt in x- and z-direction are denoted as horizontal velocity u and vertical velocity w, respectively.

The boundary conditions at the surface of the heat source $(z = 0)$ are

$$u(x, 0) = w(x, 0) = 0, \quad T(x, 0) = T_w(x) \tag{4}$$

By definition, the interface condition at the phase interface $(z = \delta(x))$ is given by the melting temperature

$$T(x, \delta(x)) = T_m \tag{5}$$

The other boundary conditions at the phase interface are no-slip condition, melt inflow and the Stefan condition. These can be simplified assuming phase-independent densities $\rho_S = \rho_L$ [13] and $U(x)\, n_z \approx U(x)$, where n_z is the z-component of the unit normal at the phase interface

$$u(x, \delta(x)) = 0, \quad w(x, \delta(x)) = -U(x), \tag{6}$$