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Thermo-mechanical analysis of periodic porous materials with microscale heat transfer by multiscale asymptotic expansion method

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ABSTRACT

A novel multiscale asymptotic method used to simulate thermo-mechanical analysis of periodic porous materials with microscale heat transfer is systematically studied. In these materials, heat radiation and heat convection that account for the scale effect of unit cells have an important impact on the macroscopic temperature and stress fields, which is our particular interest in this study. The scale effect is thought to be the result of microscopic heat transfer, the amount of which depends on the microscale pore size of porous materials. The higher-order multiscale formulations for computing the dynamic thermo-mechanical coupling problem with the inertia term, coupling term, convection term and radiation term are given successively. Then, the corresponding numerical algorithm based on the finite element-difference method is brought forward in details. Finally, numerical examples are given to demonstrate the efficiency and validity of the proposed method. The results indicate the disadvantages of classical finite element method.

1. Introduction

Porous materials have many elegant qualities, such as low relative density, heat insulation, high heat resistance, etc, and have been widely used in a variety of industrial products. These kinds of materials have small pores that may be open, connected or closed and often subjected to thermal or chemical influence at the internal and the external boundaries. Especially, porous materials designed as insulation for thermal protection system (TPS) have attracted tremendous attention and wide research interest in practical engineering applications. During spacecraft's flying out or reentry into the atmosphere, its surface will bear strong aerodynamic force and heat. Inevitably, the effect of heat radiation and convection should not be omitted, and the thermo-mechanical coupling behavior of porous materials would become more complicated. Besides, as the materials often have periodic configurations and characteristic coefficients oscillate rapidly in small cells, it is necessary to develop a new kind of effective numerical method for predicting the physical and mechanical performance of the porous materials.

Heat transfer in porous materials contains conduction, convection and radiation. Radiation is a way of heat transmission and plays an important role in heat transfer at high temperature. Considering the interior surface radiation of periodic porous materials, some interesting works were introduced to the determination of radiative properties in the past years [1–6]. Convection occurs by flow, and can be neglected at low pressure or in closed-cell porous materials [2,7]. However, it manifests significantly in heat transfer as the pores are large or connected [8]. El Ganaoui [9] extended homogenization method to a linear heat conduction problem with rapidly varying coefficients, and the heat convection that account for the scale effect of unit cells was considered. Yang et al. [10] discussed the coupled conduction, convection and radiation heat transfer problem with small parameter ε, and obtained higher-order expansions of the solution for the problem. Allaire and Habibi [11] studied the heat conduction model with ε -order convection boundary conditions by two-scale asymptotic expansion, and gave convergence results of the asymptotic homogenization for the linearized equation of the original problem. In addition, the scale effect induced by the pore size in heat transfer and mass diffusion of porous materials were pointed out in the literature [12–14]. Thermo-mechanical coupling behavior of composites is characterized by the hyperbolic equations of motion and a parabolic heat equation, which has attracted extensive research interest from scientists and engineers [15–22]. In these work the authors considered, the thermo-mechanical coupling problem on the composite materials with rapidly varying coefficients and

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neglected the coupling term to simplify the problem. Recently, Temizer and Wriggers [23] presented a survey of the known mathematical results of the homogenization method and the multiscale approach for the linear thermoelasticity. Wan et al. [24] developed a multiscale asymptotic expansion for dynamic thermo-mechanical coupling problem in composite materials and derived the convergence results. Yang et al. [25] studied the thermo-mechanical coupling problem of porous materials with interior surface radiation by asymptotic expansion method. Terada et al. [26] discussed the scale effect on the heat transfer characteristics of unit cells and derived formal expansions for thermo-mechanical coupling problem with small parameter ε, however heat radiation behavior of the porous materials was not introduced. Meanwhile, based on higher-order asymptotic expansion, Han et al. [27] applied multiscale asymptotic expansion for static thermo-mechanical problem of functionally graded materials, which is capable of accurately estimating the effective elastic modulus, local stress and strain field. Later, Wan [28] and Yang and Cui [29] investigated the dynamic thermo-mechanical coupling problems with small periodic ε by second-order two-scale method, and gave the detailed algorithm procedure. The numerical examples in [24,25,27–29] clearly confirmed that it is important to include the higher-order corrector terms. Also from these works mentioned above it can be concluded that homogenization techniques allow sufficiently accurate predictions of effective thermo-mechanical properties of arbitrarily complex microstructures. Moreover, such techniques permit calculation of the stress and temperature fields at the microscopic scale.

Different from the previous works, the dynamic thermo-mechanical coupling performance with microscale heat transfer arising from periodic porous materials will be discussed in this paper. Particular attention is focused on the treatment of heat radiation and heat convection that account for the scale effect of unit cells. In this context, an extremely fine spatial discretization mesh is necessary for the direct numerical simulation of porous materials to capture the effect of microscale heterogeneities, and thus a prohibitive amount of computation time exceeds the ability of a general computer, even for the supercomputer. An effective way to overcome this difficulty is to develop homogenization method that can be used to obtain the equivalent material parameters as well as evaluate the actual temperature/stress field in a microscale from the macroscopic responses, which cannot only save the computational resources but also ensure the calculation accuracy [30]. Also other advantages of the method developed on science and engineering are summarized in the review paper [31]. Based on the traditional homogenization methods [32,33], various multiscale approaches for composite materials with rapidly oscillating coefficients have been proposed, refer to Refs.[1,26,30,34–36]. However, they only considered the first-order asymptotic expansions. In some cases [4–6,11,24,25,29], the homogenized solution and first-order solution are not enough to capture the local fluctuation in some physical and mechanical fields, hence it is necessary to find higher-order multiscale asymptotic expansion for the solutions. Cao [37] proposed second order correctors in periodic homogenization applied to a elastic problem of composite materials. Cui et al. [38–41] introduced a higher-order multiscale method to predict the physical and mechanical properties of composite materials, and solved some practical engineering problems. By higher-order correctors, the microscopic fluctuation of physical and mechanical behavior inside the materials can be acquired more accurately.

In this paper, we mainly investigate the dynamic coupled problem considering both the inertia term, coupling term, convection term and radiation term in a solid domain of periodically porous materials. Heat convection occurs in the parallel thin pipes, and radiation is modeled by a nonlinear boundary condition on the pipes' walls and the surfaces of cavities. We introduce the higher-order correction terms into the asymptotic expansion of the temperature and displacement fields, and then derive a family of cell problems. A newly multiscale asymptotic method with less effort and computational cost for simulating the dynamic thermo-mechanical coupling behavior is proposed. Finally, some significant examples are computed to show that the results obtained by the multiscale method developed in this paper agree well with the reference solutions for thermo-mechanical analysis of periodic porous materials. Meanwhile, the CPU time and memory storage can be reduced drastically.

The remainder of this paper is outlined as follows. The detailed governing equations of thermo-mechanical problem are given in Section 2. Section 3 is devoted to the formulations of the multiscale asymptotic expansion for the thermo-mechanical coupling problems. In Section 4, the multiscale finite element algorithms are given in details. In Section 5, the corresponding numerical results for the nonlinear coupled problem obtained by the proposed method, are shown, which demonstrate the multiscale analysis method is effective. Finally, some conclusions are presented in Section 6.

Throughout the paper the Einstein summation convention on repeated indices is adopted.

2. Governing equations of thermo-mechanical problem

It is well known that under the conditions of thermo-mechanical coupling analysis problem for the porous materials with small periodic configuration leads to the following problem (see Refs. [21,23–29]):

$$\begin{aligned}&\rho^\varepsilon(x)\frac{\partial^2 u_{\varepsilon i}(x,t)}{\partial t^2}+\frac{\partial}{\partial x_j}(\beta^\varepsilon_{ij}(x)(T_\varepsilon(x,t)-\bar{T}))-\frac{\partial}{\partial x_j}(a^\varepsilon_{ijkl}(x)e_{kl}(\mathbf{u}_\varepsilon(x,t)))\\&\quad=f_i(x,t),\quad (x,t)\in\Omega^\varepsilon\times(0,t_*),\ i=1,2,3,\\[2ex]&\rho^\varepsilon(x)c^\varepsilon(x)\frac{\partial T_\varepsilon(x,t)}{\partial t}+\bar{T}\beta^\varepsilon_{ij}(x)\frac{\partial}{\partial t}\left(\frac{\partial u_{\varepsilon i}(x,t)}{\partial x_j}\right)\\&\quad-\frac{\partial}{\partial x_i}\left(k^\varepsilon_{ij}(x)\frac{\partial T_\varepsilon(x,t)}{\partial x_j}\right)=g(x,t),\quad (x,t)\in\Omega^\varepsilon\times(0,t_*),\end{aligned}\tag{1}$$

where $T_\varepsilon(x,t)$ and $\mathbf{u}_\varepsilon(x,t)=(u_{\varepsilon 1}(x,t),u_{\varepsilon 2}(x,t),u_{\varepsilon 3}(x,t))^T$ denote the temperature and displacement field; $k^\varepsilon_{ij}(x)$, $a^\varepsilon_{ijkl}(x)$ and $\beta^\varepsilon_{ij}(x)$ $(i,j,k,l=1,2,3)$ the coefficients of thermal conductivity tensor, stiffness tensor and thermal modulus tensor; $\rho^\varepsilon(x)$ and $c^\varepsilon(x)$ the density and specific heat; $g(x,t)$ and $\mathbf{f}(x,t)=(f_1(x,t),f_2(x,t),f_3(x,t))^T$ the internal thermal source and body force; $\bar{T}$ the reference temperature. $e_{kl}(\mathbf{u}_\varepsilon(x,t))$ is the strain evaluated from displacement $\mathbf{u}_\varepsilon(x,t)$ such that

$$e_{kl}(\mathbf{u}_\varepsilon(x,t))=\frac{1}{2}\left(\frac{\partial u_{\varepsilon k}(x,t)}{\partial x_l}+\frac{\partial u_{\varepsilon l}(x,t)}{\partial x_k}\right).$$

The boundary conditions are specified as follows:

$$\begin{aligned}&T_\varepsilon(x,t)=\bar{\bar{T}},\quad (x,t)\in\Gamma_T\times(0,t_*),\\&\mathbf{u}_\varepsilon(x,t)=\bar{\bar{\mathbf{u}}},\quad (x,t)\in\Gamma_u\times(0,t_*),\\&T_\varepsilon(x,0)=T^0(x),\quad \mathbf{u}_\varepsilon(x,0)=\mathbf{u}^0(x),\quad \frac{\partial\mathbf{u}_\varepsilon(x,0)}{\partial t}=\mathbf{u}^1(x),\quad x\in\Omega^\varepsilon,\\&\upsilon_i k^\varepsilon_{ij}(x)\frac{\partial T_\varepsilon(x,t)}{\partial x_j}=0,\quad (x,t)\in\Gamma_q\times(0,\ t_*),\\&-\upsilon_j((a^\varepsilon_{ijkl}(x)e_{kl}(\mathbf{u}_\varepsilon(x,t)))-\beta^\varepsilon_{ij}(x)(T_\varepsilon(x,t)-\bar{T}))=0,\\&\quad (x,t)\in\Gamma_\sigma\times(0,\ t_*),\end{aligned}\tag{2}$$