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Laminar film condensation of pseudo-plastic non-Newtonian fluid with variable thermal conductivity on an isothermal vertical plate

Xinhui Si [a,*,1], Xiandong Zhu [a], Liancun Zheng [a], Xinxin Zhang [c], Ping Lin [a,b]

[a] School of Mathematics and Physics, University of Science and Technology, Beijing 100083, China
[b] Division of Mathematics, University of Dundee, Dundee DD1 4HN, United Kingdom
[c] School of Mechanical Engineering, University of Science and Technology, Beijing 100083, China

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ABSTRACT

In this paper, we examine the laminar film condensation of pseudo-plastic non-Newtonian fluids with variable thermal conductivity on an isothermal vertical plate. The thermal conductivity is assumed to be power-law-dependent on the velocity gradient. The dual similar solutions, which are influenced by the parameters of the power law number n and the thickness η_δ of the condensation film, are obtained numerically by Runge–Kutta method coupled with shooting method. More attention is paid to discuss the first branch of the solutions with physical meaning. Especially the effects of above both parameters on the velocity and temperature distribution, condensation mass flow rate and the local Nusselt number are analyzed in detail.

1. Introduction

The study of laminar film condensation has received great attention due to its wide applications in engineering and industry, which include the design of heat exchanger, heat and fluid flows for some industrial drying and cooling processes, enhanced recovery of petroleum resources, packed-bed heat exchangers, solidification of castings, geothermal reservoirs, and so on. The pioneering work about laminar film condensation was done by Nusselt [1], who considered the condensation onto an isothermal plate maintained at a constant temperature below the saturation temperature of the surrounding quiescent vapor. Later on, more works have been done to refine Nusselt's theory. For example, Bromley [2] and Rohsenow [3] proposed modifications to the latent heat of condensation to be used in assessing heat transfer at the plate, respectively. However, both of them neglected the inertial effects. Using boundary layer theory and similarity methods, Sparrow and Gregg [4] investigated numerically the gravity driven laminar film condensation on a vertical plate, whose work showed that the inertial effects on heat transfer are limited if the Prandtl number is larger than 10. To be more important, they firstly recognized the close parallels between natural convection boundary layers and laminar film condensation. The importance of such results has been well known and documented in Ref. [5]. Chen [6] considered the retarding effect of vapor shear stress on the condensation film by perturbation methods. A comparison of Chen's results with Sparrow and Gregg's ones shows that the influence of surface shear stress is negligible at high Prandtl number. Koh et al. [7] noticed that the effect of the shear stress is significant only when the condensation rate is sufficiently high. Rose [8] confirmed Koh's conclusion and gave a more accurate expression for the Nusselt number. Mendez et al. [9] studied the conjugate condensation-heat conduction process of a saturated vapor in contact with a vertical fin, including both longitudinal and transversal heat conduction effects.

Recently, a considerable attention has been devoted to the problem of predicting the behavior of non-Newtonian film condensation. Among these classical works, the power-law flow is a significant type for its importance and simplicity. In 1960 Schowalter [10] and Acrivos et al. [11] successfully applied the boundary layer assumptions to the power-law model. Astarita et al. [12] also conducted the fully developed laminar film flow of non-Newtonian power-law fluids along a plane surface, who measured the film thickness for various inclinations and flow rates. Later, Therien et al. [13] conducted the similar work and compared the experimental data with an analytical expression for the thickness of fully developed films of power-law fluids. Sylvester et al. [14] also

* Corresponding author. Tel.: +86 01062332589.
E-mail address: sixinhui_ustb@126.com (X. Si).
[1] Current address: Department of Mathematics, University of Science and Technology, Beijing 100083, China.

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measured the film thickness as a function of the volumetric flow rate, but they primarily focused on the onset of rippling on the film surface and the characteristics of wavy film. In addition, some theoretical analysis of the power-law fluid film have been done by means of integral method or similarity analysis [15–22].

In most of the classical works, the authors only take into account the power-law kinematic viscosity in momentum equations of non-Newtonian fluids and still treat the thermal conductivity as a constant. However, in practical situations, physical properties require to be variable. To describe the heat transfer properly, the thermal conductivity for power-law fluids was assumed to be power-law-dependence in mathematical modeling view. Pop et al. [23,24] proposed a model that the thermal conductivity of non-Newtonian fluids was power-law-dependent on the velocity gradient. Zheng et al. [25–27] also made an analogy between the velocity field and the temperature field.

Durlofsky and Brady [28] ever indicated that similarity solutions are important in helping us understand the behavior of fluids and that these solutions may not represent a physically realizable flow or stability in physics while they discussed existence of the multiple solutions in the Berman's problem. Maybe due to this reason, most of the investigations to the fluids problems with the multiple solutions, such as the fluid flow between two rotating coaxial disks [29], the boundary layer flow of fluid over a moving plate [30,31], the mixed convection flow on a vertical porous plate [32,33], the flow of fluid in a porous channel or pipe [34–38] et al., were made from the mathematical point of view. In more recently, Xu et al. [39] obtained the analytically solution of the laminar film condensation of saturated steam on an isothermal vertical plate using the homotopy analysis method. Furthermore, they found the dual solutions are obtained for a range of values of the parameter η_δ. Then the present study can be, therefore, regarded as the extension of the paper Xu et al. [39], by considering the laminar pseudo-plastic non-Newtonian fluids film condensation with variable thermal conductivity over an isothermal vertical plate. The associated transfer characteristics are discussed in some domains of parameters, including the power-law number n, the generalized Prandtl number N_{Pr} and the thickness η_δ of the film.

2. Governing equations

Consider the stead film condensation of power-law fluid on an isothermal vertical flat plate. Here we assume that the vapor reservoir is stationary and is everywhere at the saturation temperature T_{sat}, and that momentum changes within the film are negligible. The viscous shear force of the vapor on the interface of the film also is neglected. (x, y) are the cartesian coordinates downward and normal to the flat surface. u and v are the velocity components in x and y directions, respectively. The film is assumed to be thin and that the change of pressure across the film is negligible. Furthermore, the velocity gradient in the y-direction is much greater than that in x-direction. Under these assumptions, the full governing equations in the liquid phase can be reduced to the following boundary-layer equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\gamma\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right) + \left(\frac{\rho_l - \rho_v}{\rho_l}\right)g, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\alpha_1\frac{\partial T}{\partial y}\right), \tag{3}$$

where n is the power-law number, g is the gravity acceleration, and ρ_v, ρ_l are the vapor and liquid density, respectively. Here $\nu = \gamma\left|\frac{\partial u}{\partial y}\right|^{n-1} = \frac{K}{\rho_l}\left|\frac{\partial u}{\partial y}\right|^{n-1}$ is the kinematic viscosity, where K is the flow consistency index for non-Newtonian viscosity, and α_1 is the thermal diffusivity. In terms of the relationship of Prandtl number to the thermal diffusivity and kinematic viscosity, it may be seen that the thermal diffusivity should be defined as $\alpha_1 = \omega\left|\frac{\partial u}{\partial y}\right|^{n-1}$ for $\frac{\partial u}{\partial y} \neq 0$ and $\alpha_1 = \omega = \frac{k}{\rho_l c_{pl}}$ for $\frac{\partial u}{\partial y} = 0$. The case $n = 1$ corresponds to the Newtonian fluid. For $0 < n < 1$, the effective viscosity decreases with the shear rate and the behavior is shear thinning (or pseudoplastic). Conversely, $n > 1$ is the viscosity increases with the shear rate and the behavior is shear thickening (or Dilatant) [25,26].

The appropriate boundary conditions are [40]

$$y = 0 : u = 0, \quad v = 0, \quad T = T_w; \tag{4}$$

$$y = \delta : \frac{\partial u}{\partial y} = 0, \quad T = T_{sat}, \tag{5}$$

where T_w is the plate temperature. δ is the boundary layer thickness of liquid film at the position x.

In terms of the standard definition of the stream function ψ such that $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, the following similarity variables can be introduced:

$$\psi = Ax^{\alpha}f(\eta), \quad \theta(x,\eta) = \frac{T - T_{sat}}{T_w - T_{sat}}, \quad \eta = Byx^{\beta}. \tag{6}$$

where A, B, α, β are constants to be determined and $f(\eta)$ also denotes the dimensionless function. Then u and v velocity component are defined as:

$$u = \frac{\partial\psi}{\partial y} = ABx^{\alpha+\beta}f'(\eta), \quad v = -\frac{\partial\psi}{\partial x} = -Ax^{\alpha-1}(\alpha f(\eta) + \beta\eta f'(\eta)) \tag{7}$$

In general, $f'(\eta)$ is equivalent of the velocity u.

Substitute Eqs. (6) and (7) into Eqs. (1)–(3) and balance the dimension of two sides of each equation, then A, B, α and β can be determinated as follows:

$$\alpha = 1 - \frac{1}{2(n+1)}, \quad \beta = -\frac{n}{2(n+1)}, \tag{8}$$

$$AB = \left(\frac{\rho_l - \rho_v}{\rho_l}g\right)^{\frac{1}{2}}, \quad B = \left(\frac{\rho_l - \rho_v}{\rho_l}g\right)^{\frac{2-n}{2(n+1)}}\left(\frac{1}{\gamma}\right)^{\frac{1}{n+1}}. \tag{9}$$

The governing Eqs. (1)–(3) can be transformed into the following equations

$$\left(|f''(\eta)|^{n-1}f''(\eta)\right)' - \frac{1}{2}f'^2(\eta) + \frac{2n+1}{2(n+1)}f(\eta)f''(\eta) + 1 = 0, \tag{10}$$

$$\left(|f''(\eta)|^{n-1}\theta'(\eta)\right)' + N_{Pr}\frac{2n+1}{2(n+1)}f(\eta)\theta'(\eta) = 0, \tag{11}$$

with the corresponding boundary conditions

$$f(0) = 0, \quad f'(0) = 0, \quad \theta(0) = 1, \tag{12}$$

$$f''(\eta_\delta) = 0, \quad \theta(\eta_\delta) = 0, \tag{13}$$

where $N_{Pr} = \frac{\gamma}{\omega}$ is the generalized Prandtl number, η_δ is the value of η at the outer edge of the film. It should be noted that $\eta_\delta = B\delta x^{-\frac{n}{2n+2}}$ varies in the range $0 < \eta_\delta < 2$ in the most text books and papers (such as [4]). Here we also follow the same assumption.

2.1. The velocity distribution, mass and heat transfer in the film

Similar to Oosthuizen's work [40], we consider a control volume with unit width. According to the conservation of momentum, the forces on the control volume can be balanced.