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Study on emptying-box problem with different inflow directions

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ABSTRACT

This study investigated the emptying-box problem with horizontal and vertical inflow directions. The salt-bath technique was used in simulation experiments with an acrylic reduced-scale model. The salt-bath technique uses salt water and fresh water to simulate the density difference between inside and outside fluids. The light-attenuation technique was used to analyze the experimental intensity data on the transient stratified flow. The experiments were categorized into two series based on the inflow direction: the horizontal inflow type emptying box EM(H) and the vertical inflow type emptying box EM(V). The emptying process of EM(H) was observed to only involve emptying the dense layer, similar to the classical displacement flow. The emptying process of EM(V) included emptying the dense layer and mixed layer. The results showed that the emptying times of EM(H) and EM(V) decreased when either the total effective opening area or initial reduced gravity was increased. The time to empty the dense layer in the EM(V) series was about 50–60% of that in the EM(H) series, and the total emptying time was much longer for the former series than for the latter series under the same conditions. Previous theoretical models were applied to both series and showed reasonable agreement with the experimental results. A new modified theoretical model for EM(V) was developed and compared with the previous model. The new model demonstrated improved accuracy with regard to estimating the experimental evolution.

1. Introduction

The emptying-box problem [1] is an extension of the filling-box problem [2], and both problems often arise in industry and nature. The emptying-box problem involves utilizing the buoyancy force between the inside and outside fluids to empty the fluid in the box. The problem is characterized as follows: the density difference between the inside and outside fluids is small compared to the density of the inside or outside fluid, and the two fluids are miscible.

Natural ventilation is usually classified as wind-driven or buoyancy-driven depending on the driving forces. Wind-driven ventilation is driven by the inertial wind force from the exterior environment. Buoyancy-driven ventilation is driven by the buoyancy force due to the density or temperature difference between the interior and exterior fluids. The emptying-box model has previously been applied to many buoyancy-driven ventilation designs.

Transient natural ventilation flows have received increasing attention in recent research. For example, Coffey and Hunt [3] classified the flow field of the emptying-box problem into four types by using the initial top-opening Froude number $Fr_T(0) = 0.67$ and the initial bottom opening Froude number $Fr_B(0) = 0.33$ as two critical values. The four types are denoted as the unidirectional classical Displacement flow, Displacement flow with interfacial mixing, bidirectional Exchange flow, and Exchange flow with interfacial mixing. According to Hunt and Coffey [4], the two Froude numbers at the top and bottom openings are determined by three geometric parameters: the fractional initial dense layer depth, the ratio of the top opening area to the bottom opening area, and the characteristic length scale of the top or bottom opening relative to the initial dense layer depth.

This paper presents an analysis of emptying-box problems with horizontal and vertical inflow directions. This research focused on the effect of the inflow direction on emptying-box problems, the preliminary results were presented by Lin and Fan [5]. In Section 2, the theoretical model developed by Linden et al. [1] for an emptying-box problem with a classical displacement flow is rephrased and applied to the case with a horizontal inflow direction, or EM(H). The proposed modifications to the theoretical model developed originally by Coffey and Hunt [3] for the emptying-box problem of a displacement flow with interfacial mixing are presented in this paper. Both the new modified and original models are used for the case with a vertical inflow direction, or EM(V). Section 3 presents the experimental setup,

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Nomenclature

A^*	total effective area (m^2)
a_x	opening area at the location x (m^2)
b_j	radius of the jet flow (m)
b_l	radius of the jet flow at h_l (m)
C_d	discharge coefficient (–)
C	energy loss coefficient (–)
g'	reduced gravity ($m\,s^{-2}$)
H	total height of the space (m)
h	interface level (m)
h_l	dense layer interface level (m)
I	light intensity (–)
i	pixel index (–)
L	distance away from the jet flow virtual origin (m)
M_T	momentum flux at the top opening ($m^4\,s^{-2}$)
Q	volumetric flow rate ($m^3\,s^{-1}$)
Q^*	penetrative entrainment volumetric flow rate ($m^3\,s^{-1}$)
Q_j	volumetric flow rate of the jet flow ($m^3\,s^{-1}$)
S	cross-section area (m^2)
t	time (s)
w_j	average vertical velocity of the jet flow ($m\,s^{-1}$)
w_l	average vertical velocity of the jet flow at h_l ($m\,s^{-1}$)
x	horizontal coordinate (m)
z	vertical coordinate (m)
z_v	virtual origin height correction (m)

Dimensionless parameters

Fr_x	Froude number for the opening x (–)
R	opening area ratio ($=\frac{a_T}{a_B}$)
$\hat{z}_v$	dimensionless virtual origin height correction ($=\frac{z_v}{h_l(0)}$)
λ_x	initial Richardson number through the opening ($=\frac{\sqrt{a_x}}{h_l(0)}$)
ξ_0	fractional initial dense layer depth ($=\frac{h_l(0)}{H}$)

Greek symbols

α	empirical constant for jet velocity profile (–)
β	empirical constant for jet radius profile (–)
Δ	magnitude of the difference (–)
ρ	density ($kg\,m^{-3}$)

Subscripts

B	bottom opening
E	totally empty
ex	experimental result
jet	jet with Gaussian velocity profile
l	dense brine layer
m	mixed layer
n	new modified model
T	top opening
t	jet with the top-hat velocity profile
th	theoretical prediction

two series of experiments and the data analysis approach. The experimental results are presented and compared with the theoretical predictions in Section 4. The conclusions of this study are given in Section 5.

2. Theoretical analysis

The theoretical analysis is presented in two parts: problems with a (1) horizontal inflow direction (EM(H)) and (2) vertical inflow direction (EM(V)). Both parts focus on the transient state for some certain initial conditions. The theoretical model on the classical displacement flow was developed by Linden et al. [1], and an analytical solution is given in Section 2.1. The classical displacement flow model is applied to EM(H). The theoretical model of EM(V) was originally developed by Coffey and Hunt [3] and needs to use a numerical algorithm to obtain a solution. Section 2.2 presents some proposed modifications to the original theoretical model of EM(V). Fig. 1 shows the height coordinate used in this paper. The origin is at the left bottom corner of the tank, as shown in Fig. 2.

2.1. Emptying box with horizontal inflow

The theoretical model of EM(H) shown in Fig. 1(a) is based on the volume conservation equation and the assumption of buoyancy-driven flow without interfacial mixing. The flow rate of the box Q, the time-dependent interface level between the fresh and dense layers $h_l(t)$, and the time to empty the dense layer t_l, are given as follows:

$$Q(t) = A^*({g_l}' h_l(t))^{\frac{1}{2}}, \tag{1}$$

$$h_l(t) = \left(h_l(0)^{\frac{1}{2}} - \frac{A^*}{2S} {g_l'}^{\frac{1}{2}} t\right)^2 \tag{2}$$

and

$$t_l = \left(\frac{2S}{A^*}\right)\left(\frac{h_l(0)}{g_l'}\right)^{\frac{1}{2}}, \tag{3}$$

where A^* is the total effective area of the box, g_l' is the reduced gravity of the dense layer, $h_l(0)$ is the initial interface height of the dense layer, and S is the cross-sectional area. The total effective area A^*, is related to the areas of top and bottom openings (a_T and a_B) as follows:

$$A^* = \left(\frac{1}{2a_T^2 C_d^2} + \frac{1}{2a_B^2 C_d^2}\right)^{-\frac{1}{2}}. \tag{4}$$

The discharge coefficient (refer to Ref. [6]), $C_d = 0.61$, was applied to the theoretical predictions in this research.

2.2. Emptying box with vertical inflow

Fig. 1(b) show a schematic for the theoretical model of EM(V). The volume conservation equation for the dense layer is given by

$$S\frac{dh_l}{dt} = -(Q + Q^*), \tag{5}$$

where Q^* is the flow rate entrained from the dense layer.

The volume conservation equation for the mixed layer is given by

$$S\frac{d}{dt}(h_m - h_l) = Q_j(h_m) + Q^*, \tag{6}$$

where h_m is the interface level between the fresh and mixed layers, and $Q_j(h_m)$ is the flow rate of the inflow jet at h_m.

The buoyancy flux conservation of mixed layer is given by

$$S\frac{d}{dt}[g_m'(h_m - h_l)] = Q^* g_l', \tag{7}$$

where g_m' is the reduced gravity of the mixed layer.