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Impurity transport in developed Rayleigh–Bénard convection

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ABSTRACT

A model similar to the dual-porosity model is proposed, which describes impurity transport in developed Rayleigh–Bénard convection. The transport characteristics are considered for the two flow configurations (a chain of rolls and a lattice of hexagons) and for a wide interval of Rayleigh number values (when the flow is steady or time-dependent). For the chain of rolls subdiffusion at small times and the classical diffusion with effective diffusivity at large times are described. Concentration profile at distances much greater than the impurity cloud size is evaluated. Dependence of effective diffusivity on characteristics of flow fluctuations at large Rayleigh number is considered.

1. Introduction

The transport of passive impurity in developed Rayleigh–Bénard (RB) convection in fluid layer heated from below attracts attention of investigators for several decades. This problem is actual both for a layer of a free fluid and for saturated porous media, and is of theoretical and practical importance in diverse fields of science and engineering [1].

Convective flows in a free fluid layer and in a layer of saturated porous medium are to a large extent similar and depend on the value of Rayleigh number R. The definition of R for two cases is somewhat different. For the layer of free fluid $R = \frac{g\beta\Delta T H^3}{\chi\nu}$, where g is the acceleration due to gravity, β is the coefficient of thermal expansion of the fluid, ΔT is the temperature differential across the region, H is the depth of the layer, χ is the thermal diffusivity of the fluid and ν is its kinematic viscosity. For the layer of saturated porous medium Rayleigh number is usually defined as $R = \frac{g\beta\Delta T k H}{\chi\nu}$, where k is the permeability of the porous medium.

At small R the fluid is motionless, so that heat transport is determined only by thermal conductivity (over fluid or over fluid and porous matrix), and impurity transport occurs due to molecular diffusion in solute.

When R exceeds some critical value, the state of rest of the fluid becomes unsteady. The critical value for free fluid is of the order of $R_1 \approx 1700$, and depends on layer geometry and on boundary conditions, while for porous medium $R_1' = 4\pi^2$. As a result, at $R > R_1$ convective cells are formed, which facilitate heat transport from the bottom to the top of the layer. Depending on the layer geometry, boundary conditions on the layer's surfaces, Prandtl number, temperature dependence of thermal diffusivity, the influence of surface tension on the process, the development of two types of cells are possible: (1) convective rolls, and (2) hexagons. Fluid motion in these cells (and, hence, impurity advection) takes place along closed streamlines on the scales of the order of layer's depth. Nevertheless, the formation of such stable periodic closed cells leads to the acceleration of impurity transport along layer's plane.

Further enhancement of Rayleigh number leads to fluctuations of the flow, while the temporarily averaged flow velocity field retains the form of closed streamlines as for steady flow [2]. Experimental studies of time-dependent RB convection for the case of rolls has been carried out in [2], and it was shown that for free fluid layer the fluctuating regime begins at $R_2 \approx 19R_1$. At greater values of Raleigh number fluid motion becomes turbulent. In porous medium flow fluctuations for closed circulating flow were studied by means of Hele Shaw cell [3] only for one vortex. For hexagons flow fluctuations in saturated porous medium has been qualitatively described in [4].

If an impurity is present in liquid then a problem arises how the above described flow will influence the impurity transport.

Impurity transport in RB convection was studied experimentally and theoretically for the free fluid layer (see [5] and the literature herein). Main attention of investigators was paid to the transport along the chain of rolls in the condition of the steady flow. It was shown that a characteristic time τ_1 exits, so that at times larger than τ_1 the transport along the chain (in the direction perpendicular to the rolls axis) is described by the classical diffusion with the effective diffusivity

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$$D_{eff} \approx d\sqrt{Pe}. \qquad (1)$$

Here d is molecular diffusivity of impurity in solute and $Pe = \frac{VH}{d}$ is the Peclet number determined by characteristic value of the fluid velocity V at the rim of rolls. Impurity transport in the direction of roll axis is naturally determined by molecular diffusion. At times smaller than τ_1 it was supposed (see, e.g., [6]), that the transport would be described by non-classical regime. Here dispersion of impurity cloud along roll chain grows according to

$$\sigma^2 \sim t^{\gamma}, \qquad (2)$$

with $\gamma < 1$. According to the results of theoretical work [6], in the case of free fluid for the exponent γ one should expect $\gamma = 2/3$. To verify this an experiment was carried out [5] in which impurity transport in a chain of vortices was investigated. These vortices were formed by flowing of electric current through liquid placed into spatially modulated magnetic field. The results obtained indicate the presence of time interval, where non classical (subdiffusive) transport regime occurred. But, from our point of view, due to the experimental arrangement and according to results presented (see Fig. 6 of the work [5]) the validity of the law $\sigma^2 \sim t^{2/3}$ for the case of impurity transport along the chain of rolls in RB convection is rather questionable.

Experimental studies of impurity transport in RB convection of free fluid was carried out in the work [2] for the interval of Rayleigh numbers when the flow (in the form of rolls) becomes fluctuating but the temporarily averaged structure of the flow remains as for steady flow. Here characteristics of the flow and effective diffusivity for impurity transport have been measured. To describe an increase of effective diffusivity in this case a numerical model was also proposed here which described the observed impurity transport characteristics basing on the failure of closed character of rim streamlines.

Impurity transport in the condition of RB convection in saturated porous medium was discussed in the theoretical work [7] in which two time intervals were predicted basing on scaling relations for migration of one particle. According to the results of [7] one should expect the classical diffusion with effective diffusivity (1) at large times, and subdiffusion (2) with $\gamma = 1/2$ at small times.

Impurity transport in patterns with hexagons (both for free fluid and for porous media) appears not to be investigated experimentally up to now. It should be noted assertion pointed in [8], that in this case one may expect much smaller increase of the effective diffusivity, namely, $D_{eff} \approx d \ln Pe$.

The aim of the present work is to develop the dual-porosity model (DPM) to describe a passive impurity transport in the developed RB convection both for a free fluid flow and for the flow in porous medium. Two types of impurity will be considered: (1) the dissolved matter that follows the laws of molecular diffusion and (2) the particles that are macroscopic enough to be subject to Brownian motion. The main body of the paper is devoted to the first case (dissolved impurity), and the transport of macroscopic particles is analyzed in the last section. It should be stressed that application of DPM is not connected with the presence of two types of porosity in the medium. The term dual porosity means the presence of two different subsystems of particles determined by flow structure which possess different transport properties. Taking into account that the flow structure is similar for both cases, the results will be also applicable for free fluid and for flow in porous medium as well.

Below the paper is organized as follows. In Section 2 impurity transport in a chain of rolls is considered for the values of Rayleigh number when the flow is stable. A model for a dissolved impurity will be proposed and transport regimes will be considered. The behavior of impurity concentration at asymptotically large distances from the source will be also described. In Section 3 basing on the developed model the transport process will be analyzed at the values of R when essential fluctuations of flow velocity arise but temporarily averaged structure of the flow (the chain of rolls) keeps its form. In the next section the transport in a periodic system of hexagons is considered. In Section 5 a possible influence of flow fluctuations in hexagons on impurity transport is analyzed. Section 6 is devoted to application of the model to macroscopic particle transport. In the last section the main conclusions are presented.

2. Impurity transport in steady convection rolls

The transport of dissolved impurity in the developed natural convective flow in the form of a chain of rolls is considered. We will be interested in the transport in the direction along the chain (in the direction along roll axis usual diffusion with molecular diffusivity d takes place). Impurity migration is determined by: (1) advection along closed streamlines inside each roll, and (2) diffusion across streamlines at boundaries between two rolls (see Fig. 1). Transport due to diffusion along streamlines can be neglected in comparison with the advection.

It is naturally to believe that streamlines with maximal velocity (denote it as V) are localized in the vicinity of the roll rim [6]. Their exact position depends on the boundary conditions, but for our model it is unessential. It is important, however, that these streamlines are situated in immediate proximity to the boundary between two rolls.

Passing along vertical boundary of two neighboring rolls (from point 1 to point 2 in Fig. 1) an impurity molecule (below we will call it "particle" for short) can diffuse from one roll into another.

A passage time of the particle along the boundary between two rolls is

$$\tau_c \approx H/V. \qquad (3)$$

It determines a characteristic thickness of boundary layer (stream tube at the roll rim) w, from which the particle can pass into the neighboring cell. Taking into account that the diffusion time over scale w is

$$\tau_d \approx w^2/d, \qquad (4)$$

from the equality $\tau_d \approx \tau_c$ one obtains

$$w \approx \sqrt{d\tau_c} \approx \sqrt{\frac{dH}{V}} = H \cdot Pe^{-\frac{1}{2}}. \qquad (5)$$

Here $Pe = \frac{HV}{d}$ is Peclet number which was also introduced earlier.

We consider the case $Pe \gg 1$ that, in turn, determines the lower limit of Rayleigh number for which the model is applicable. From here it is easy to see that $w \ll H$.

The process of particle transition from one roll to another is random. The particles from boundary layer of the thickness w during one passage along the boundary between two cells due to diffusion may both cross the boundary or remain in original roll. In that later case during their motion along the stream tube they may both remain in this tube or to pass towards the interior of the roll. Here we consider the case when the boundaries at the top and at the bottom of the layer are impenetrable. The remaining in the tube particles are transported to the opposite boundary. Here the process of particle transition to the neighboring cell can repeat, again, in a random manner.

As a result, at scales considerably exceeding the roll size H the particles in the stream tubes of the boundary layers of the thickness w make random jumps to the right and to the left at length H with frequency τ_c^{-1}. It can be described as one-dimensional (along the roll chain) diffusion with the diffusivity