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New Fourier-series-based analytical solution to the conduction–convection equation to calculate soil temperature, determine soil thermal properties, or estimate water flux

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ABSTRACT

Temperature is an important physical variable of soil. Heat transfer in soils predominantly occurs due to conduction and convection. In this paper, we present a new analytical solution, based upon the Fourier series boundary conditions for soil surface temperature, in which the separation of variables for the heat conduction–convection equation was established. Data that had been collected from the Qinghai-Xizang (Tibet) Plateau (QXP) were used to calculate the thermal diffusivity and the liquid water flux density using different methods. The results of the soil thermal diffusivity using 5 cm as the upper boundary for the soil depth were used to calculate the soil temperature using both the single sine wave conduction and conduction–convection model and the Fourier series conduction–convection model. These results were then compared with the values for the temperature of the field soil measured at a depth of 10 cm. The average standard error of the estimate (SEE), the normalized standard error (NSEE) and Bias were 0.16 °C, 2.70% and 0.11 °C for the Fourier series conduction–convection method. The results indicate that the Fourier series model provides a better estimate of observed field temperatures than the sine wave model. This method provides a useful tool for determining soil thermal parameters, simulating soil temperature and the parameterization of land surface processes for modeling permafrost changes under global warming conditions.

1. Introduction

Soil temperature is an important element in land surface processes and also plays a critical role in energy balance applications; including land surface modeling, numerical weather forecasting and climate prediction [1]. Previous work has demonstrated that the response of soil temperatures to atmospheric climate change can be complicated [2]. Soil temperature also affects the physical and chemical properties of soil as well as other biochemical processes, and thus it could have an effect on biochemical processes such as plant growth [2]. Considering its important role in the above processes, it is necessary to accurately calculate soil temperature [3]. Numerous numerical atmospheric models simulating soil temperature have been developed; however, these models have only taken into consideration the process of thermal diffusion, which is a result of temperature differences and heat conduction. Neglecting heat advection associated with water flux can yield inaccurate estimations of soil temperature during the day and night [4,5]. These inaccuracies are possibly due to the neglecting of the impact of the vertical migration of moisture on soil temperature [6–8]. Therefore, it is necessary to analytically solve the simultaneous heat and mass transfer problem for improving our understanding of coupled heat- and water-flow processes.

The thermal conductivity, thermal diffusivity, and volumetric heat capacity are three important soil thermal properties [9]. These thermodynamic properties are basic parameters for simulating soil temperature. Considering that soil thermal conductivity and the thermal diffusivity are related with the soil volumetric heat capacity, only soil thermal conductivity or thermal diffusivity needs to be determined. The thermal diffusivity is typically estimated because the soil thermal diffusivity describes the transient process of heat conduction. Knowledge of this parameter not only allows for an understanding of the soil thermal properties, but also provides one of the necessary parameters for the simulation of soil temperature and heat flux [10].

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The observed soil temperature can be used to determine the soil thermal diffusivity in a variety of ways, most of which depend on the assumption that the soil is a semi-infinite medium with a constant thermal diffusivity and that the upper thermal boundary can be characterized by a harmonic function [11,12]. The main methods that have been used include the amplitude method, phase method, arctangent method, logarithmic method, numerical method and harmonic method [13]. Several of these methods have been evaluated under the assumption that the temperature at the upper boundary was well described by a sinusoidal function or by a Fourier series [14]. The results from this study found the Fourier series method to be the most reliable.

In many soils, vertical water flux can influence the soil temperature, and thus solutions to the conduction–convection equation have been applied. Several studies (e.g. [15]) have shown that temperature determined from solutions to the conduction–convection equation have compared favorably to measured multi-depth soil temperatures. Further investigations indicated that changes in soil temperature were related to the soil thermal conductivity and the soil thermal convection caused by the vertical movement of liquid [16–18]. The results indicated that the thermal conduction–convection equation had an increased capability to accurately describe the soil heat transfer processes in comparison to the classical conduction method [19].

Previous research suggests that convection heat transfer makes significant contributions to daily subsurface temperature oscillations [20]. In actuality, the diurnal change in soil surface temperature does not strictly follow a single sinusoidal curve. The use of a Fourier series to accurately describe diurnal variations in the surface soil temperature can reduce the errors due to the assumption that the temperature at the soil surface follows a single sinusoidal wave [21], but the previous method with Fourier series requires the initial soil temperature profile and the infiltrating water parameters, which would complicate its use in numerical models. Additionally, there is little work on the soil thermal parameters on the Qinghai-Xizang Plateau (QXP) [22,23]. Despite that the QXP plays a very important role in China and the global climate system [24–26], there are relatively few monitoring stations on the QXP because of the harsh environment. Therefore, calculation methods for the soil temperature and the effect of the vertical movement of soil water on the soil thermal properties of the QXP require further investigation. In this study, the Fourier series thermal conduction–convection equation was solved using the variable separation approach. Then, the influence that the vertical movement of the soil water has on the soil thermal characterises in the QXP was examined. The objectives of the present study were to (i) propose a new method for solving the thermal conduction–convection equation using the Fourier series boundary; (ii) to calculate soil thermal diffusivity and water flux density through the use of the genetic algorithm method; (iii) to compare field-measured soil temperature values with those calculated with analytical solutions using the conduction–convection model in QXP.

2. Field experiments

The Tanggula test site (33°04′N and 91°56′E), which was established in June of 2004, has an altitude of 5100 m and is situated on a gentle slope at the Tanggula Mountain Pass on of the Qinghai-Xizang Plateau in China. The monitoring site (Fig. 1) was set up on a southwest-facing gentle slope at the eastern bank of the Dangqu River near the Qinghai-Tibet High Way (QTH) in the Tanggula region. The site was approximately 1 km from the river [27], which is located in the continuous permafrost zone and belongs to the alpine grassland steppe. The vegetation in this region consists of an alpine meadow that is distributed in clusters with a coverage of about 20–30% and a height of less than 10 cm.

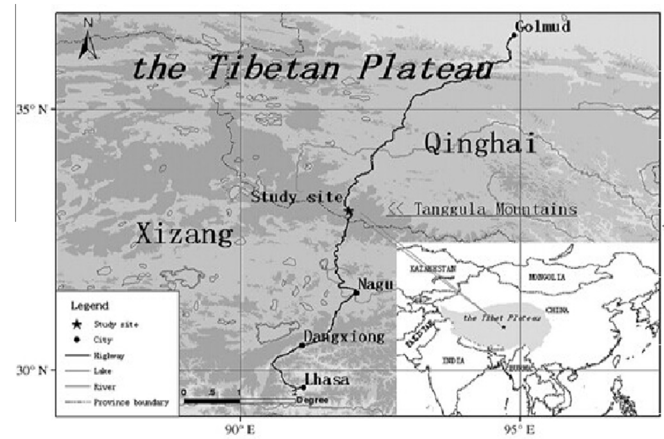

Fig. 1. Location of the Tanggula comprehensive observation site on the Tibetan Plateau in China.

The hourly meteorological data, consisting of precipitation, global radiation, net radiation, air temperature, wind speed and relative humidity, were recorded by CR23X data acquisition instrument (Campbell Scientific Inc.). The soil temperature and moisture in the active layer were recorded every 0.5 h by CR1000 data acquisition instrument (Campbell Scientific Inc.) at different depths [28]. The data used in this study were collected at the Tanggula site from September 1st, 2012 to September 5th, 2012 during an intensive observation and sunshine period in the permafrost regions of the QXP.

3. Methods

3.1. Thermal conduction equation for soil temperature (SCM)

The fundamental solution of the classic heat diffusion equation was applied to a one dimensional semi-infinite medium:

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial z^2} \tag{1}$$

where k is the thermal diffusivity (m^2 s) and $k = \lambda/Cg$; where λ is the thermal conductivity (W m^{-1}°C^{-1}) and Cg is the volumetric heat capacity of the soil (J m^{-3}°C^{-1}).

The boundary condition at a depth z_1 is given as [20]:

$$T|_{z=z_1} = \bar{T}_1 + A_1 \sin(\omega t - \phi_1), \ t \geqslant 0 \tag{2}$$

where $\bar{T}_1$ is the mean soil temperature (°C) at a depth z_1 (m), A_1 is the amplitude (°C), ϕ_1 is the initial phase (rad) and ω (7.27×10^{-5} rad/s) is the angular velocity of the Earth's rotation.

Utilizing both Eqs. (1) and (2), the soil temperature (T) at a depth z_2 is calculated as follows [29]:

$$T_{z=z_2} = \bar{T}_2 + A_1 \exp[(z_1 - z_2)\alpha] \cdot \sin[\omega t - \phi_1 - (z_2 - z_1)\alpha] \tag{3}$$

where $\alpha = \sqrt{\omega/2k}$. If we set A_2 and ϕ_2 as the amplitude and the phase at depth z_2, then:

$$A_2 = A_1 \exp[-(z_2 - z_1)\alpha]$$

$$\phi_2 = \phi_1 + (z_2 - z_1)\alpha$$

3.2. Thermal conduction–convection equation for soil temperature (SCCM)

The following conduction–convection equation for a one-dimensional unsteady soil heat transfer in the presence of a steady flow of water [7,19,30]: