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Co-current and counter-current vertical pipe moving bed heat exchangers: Analytical solutions

Pedro A. Isaza, Yuxuan Cai, W. David Warnica, Markus Bussmann *

Department of Mechanical and Industrial Engineering, University of Toronto, 5 King's College Road, Toronto, ON M5S 3G8, Canada

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ABSTRACT

Recently, analytical solutions for parallel-plate moving bed heat exchangers have been obtained by means of integral transform methods. This study extends the analysis to vertical pipe geometries due to their important industrial applicability. Steady-state energy equations, for systems operating under co- and counter-current conditions, are formulated and nondimensionalized. Laplace transforms and various forms of the expansion theorem are then used to solve the problems, resulting in temperature functions for the solids and fluid domains. Limiting cases are then analyzed and the solutions are shown to simplify to various expressions in the literature. A graphical analysis is also presented, depicting representative behaviors of the solutions and addressing their physical consistency.

1. Introduction

Over the last decade, interest has grown in the application of moving bed technologies in processes involving chemical separations, recuperation of petroleum products, drying of food materials [1], and flue gas cleaning [2]. In many of these processes, heat transfer to and from these moving beds of solids is of critical importance. Specific examples exist in the recovery of oil shale, drying of brown coal, cooling of ore cinders [3], production of nickel [4,5], and food sterilization [6]. In a very recent application, energy transfer from a moving bed of ceramics and natural stones has been explored as a cost effective candidate for delivering off-line thermal energy for steam and electricity generation [7]. This wide industrial presence drives the need for on-going investigations into moving bed transport phenomena.

A particular unit operation used for energy exchange in the above examples is the moving bed heat exchanger (MBHE). In these systems, heat is conveyed between a moving bed of particles and a secondary heating or cooling fluid. In general, MBHEs are attractive due to their low investment cost, energy consumption, and maintenance requirements [1]. Their simple design, practicality, and versatility also give them an advantage over competing technologies [3].

One particular advantage of MBHEs is their ability to accommodate different exchanger geometries including parallel-plate, shell-and-tube and double-pipe, while simultaneously allowing flow arrangements extending from counter- to co-current. For some time, thermal performance and sizing information remained empirical [8]. Attempts to solve the convection-conduction MBHE problem had been put forward [9], but an analytical solution was never identified. Only recently have solutions been presented for co-current [10] and counter-current [11] parallel-plate configurations. In these analyses, the moving solids and the secondary fluid vary in temperature as heat is exchanged between the domains.

An extension of the Cartesian analysis [10,11] to cylindrical coordinates is important since these geometries exist in shell-and-tube and double-pipe heat exchangers. These types of systems have low installation costs, ease of maintenance and cleaning, and flexibility of design [12]. Vertical pipe MBHE configurations have been previously studied [5,9,13,14], but analytical solutions remain absent. In particular, counter-flow configurations are of critical importance since they yield increased thermal gradients reducing area requirements and capital investments [15].

The objective of this work is to present the solutions to the non-homogeneous steady convection-conduction equations describing both co-current and counter-current vertical pipe MBHEs. The solutions follow the methodologies of recent parallel-plate studies [10,11]. Although these cylindrical solutions are analogous to those found in Cartesian coordinates [10,11]; the procedures for obtaining them, and the functions associated with them are quite different.

* Corresponding author. Tel.: +1 4169460690; fax: +1 4169784859.
E-mail addresses: pedro.isaza@mail.utoronto.ca (P.A. Isaza), yuxuan.cai@mail.utoronto.ca (Y. Cai), david.warnica@utoronto.ca (W.D. Warnica), bussmann@mie.utoronto.ca (M. Bussmann).

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Nomenclature

a	constant, $= NTU \cdot C$
A_{cond}	axial heat conduction area in the wall, $\pi\left(R_o^2 - R_i^2\right)$
A_{flow}	cross-sectional area of solids flow, $= \pi R_i^2$
$A_{hx,i}$	area of heat exchange based on the internal pipe radius
a_j	jth eigenvalue for a Biot number of zero
b	constant, $= \sqrt{\frac{2 \cdot Bi}{NTU}}$
Bi	Biot number, $= \frac{U_i \cdot R_i}{k_s}$
C	Capacity ratio, $= \frac{\dot{m}_s \cdot C_{ps}}{\dot{m}_f \cdot C_{pf}}$
C_{pf}	fluid specific heat capacity
C_{ps}	solids effective specific heat capacity
H	pipe height
h_o	convective heat transfer coefficient for the fluid
i	imaginary number, $= \sqrt{-1}$
j	integer number
k	number of multiple s_n roots in ψ
k_s	solids effective thermal conductivity
k_w	wall thermal conductivity
M	Axial conduction number, $= \left(\frac{k_w}{\rho_s C_{ps}}\right) \frac{(R_o^2 - R_i^2)}{R_i^2} \left(\frac{1}{u_s H}\right)$
$\dot{m}_f$	fluid mass flow rate
$\dot{m}_s$	solids mass flow rate
n	integer number, positive
NTU	Number of Transfer Units, $= \frac{U_i \cdot A_{hx,i}}{\dot{m}_s \cdot C_{ps}} = \frac{2 \cdot U_i \cdot H}{\rho_s u_s R_i C_{ps}}$
Q_{adv}	estimate of advective heat transfer rate in the solids
$Q_{wall,cond}$	estimate of axial heat conduction rate in the wall
R_i	pipe inner radius
R_o	pipe outer radius
R'_{cont}	solids/wall contact resistance
r	radial spatial coordinate
r^*	dimensionless radial spatial coordinate, $= \frac{r}{R_i}$
s	Laplace domain axial variable
s_n	simple roots of expansion theorem denominator function ψ
T_s	solids temperature function
$\overline{T_s}$	solids average temperature function
T_{si}	solids entrance temperature
$\overline{T_{so}}$	solids average outlet temperature
t_f	fluid temperature function
t_{fi}	fluid entrance temperature
t_{fo}	fluid outlet temperature
U_i	overall heat transfer coefficient based on the internal pipe radius
u_s	solids velocity
x	axial spatial coordinate
x^*	dimensionless axial spatial coordinate, $= \frac{x}{H}$

Greek letters

α_s	solids effective thermal diffusivity
ΔT_s	estimate of axial temperature difference
θ_f	dimensionless fluid temperature function, $= \frac{t_f - t_{fi}}{T_{si} - t_{fi}}$
$\theta_{f,Bi\to 0}$	dimensionless fluid temperature function for a Biot number of zero
$\theta_{f,C\to 0}$	dimensionless fluid temperature function for a capacity ratio of zero
$\theta_{f(0)}$	dimensionless fluid temperature at $x^* = 0$
$\widetilde{\theta_f}$	Laplace domain dimensionless fluid temperature function
θ_s	dimensionless solids temperature function, $= \frac{T_s - t_{fi}}{T_{si} - t_{fi}}$
$\theta_{s,Bi\to 0}$	dimensionless solids temperature function for a Biot number of zero
$\theta_{s,C\to 0}$	dimensionless solids temperature function for a capacity ratio of zero
$\widetilde{\theta_s}$	Laplace domain dimensionless solids temperature function
$\overline{\theta_s}$	dimensionless solids average temperature function
λ_n	nth eigenvalue
μ	positive eigenvalue for $C > 1$ counter-current case
ρ_f	fluid density
ρ_s	solids effective density
φ	expansion theorem numerator function
ψ	expansion theorem denominator function

In this paper, the formulation of the coupled governing energy equations, with boundary conditions, is presented, along with their nondimensionalization. Laplace transforms are applied to obtain analytical solutions for the solids and fluid temperatures. Limiting cases are then analyzed and the solutions are contrasted with various expressions in the literature. Finally, a graphical analysis explores the consistency of the solutions.

2. Model development

2.1. System description and assumptions

Consider particulate solids and a heating/cooling fluid moving co-currently or counter-currently in the vertical pipe system shown in Fig. 1. Important system dimensions include the pipe inner radius R_i, the outer radius R_o, and the height H. The solids move inside the pipe with a velocity u_s, and enter at a constant temperature T_{si}. The fluid moves with mass flow rate $\dot{m}_f$, and enters at a temperature t_{fi}.

The assumptions made in the energy model development are as follows:

1. Steady conditions, where the moving solids and interstitial fluid are assumed to be in local thermal equilibrium. As shown in previous publications, local thermal equilibrium has been found to be valid for a wide number of materials, even highly conducting metals, as long as the interstitial fluid phase is entrained [3,5,9,13,16–19] as considered here.
2. The thermo-physical properties of the solids (thermal conductivity k_s, density ρ_s, and specific heat capacity C_{ps}) are effective and constant. This assumption has been validated for vertical pipe systems for numerous porous materials [5,13,14].
3. Heat conduction in the solids occurs in the radial r-direction [9,13,14], while convective transport occurs in the axial x-direction. Transfer of energy in the solids in other directions is assumed negligible.
4. The solids move with constant velocity in the x-direction. This important assumption has been applied and validated in numerous moving bed studies for flow parallel to an adjacent wall [3,5,13,18,20].
5. The temperature of the solids at $r = 0$ is finite.
6. Convective energy transport in the fluid occurs in the x-direction only, and the thermo-physical properties (i.e. density ρ_f, specific heat capacity C_{pf}) are assumed constant.
7. A constant overall heat transfer coefficient U_i, defined on the basis of the internal area of the pipe, connects energy transport between the solids and fluid domains. It is comprised of resistances in series due to contact, wall conduction and convective transport into the fluid. In cylindrical coordinates, U_i takes the following form: