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Three-dimensional flow of nanofluid over a non-linearly stretching sheet: An application to solar energy

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ABSTRACT

This work deals with the three-dimensional flow of nanofluid over an elastic sheet stretched non-linearly in two lateral directions. Suitable boundary conditions showing the power-law variation in the velocities are imposed. Further the recently suggested model for nanofluid is considered that requires nanoparticle volume fraction at the wall to be passively rather than actively controlled. A set of similarity transformations are introduced to convert the boundary layer equations into self-similar forms. The solutions have been obtained numerically through shooting method with fourth-fifth-order Runge–Kutta integration technique. The results reveal that penetration depths of temperature and nanoparticle volume fraction are decreasing functions of the power-law index. We notice that impact of Brownian motion in the temperature and heat transfer rate from the sheet is insignificant.

1. Introduction

The flow over moving or stationary solid surfaces has been a prime interest of researchers mainly due to their variety of engineering applications. For example condensation of liquid films, drawing of filaments through a quiescent fluid, crystal growth process, metal working processes, manufacturing of plastic and rubber sheets and continuous cooling of fiber etc. Ever since the advent of boundary layer equations, various aspects of such flows have been investigated by the research community. The classical problem of two-dimensional flow induced due to non-linearly stretching surface was described by Vajravelu [1]. Cortell [2] extended this problem by considering viscous dissipation effect and variable surface temperature. Bhargava et al. [3] investigated the flow of micropolar fluid due to nonlinearly stretching sheet. Radiation and viscous dissipation effects on the two-dimensional flow above a non-linearly stretching sheet were addressed by Cortell [4]. Hayat et al. [5] derived series solutions for mixed convection flow of micropolar fluid due to stretching sheet. Kechil and Hashim [6] investigated the MHD flow over a non-linearly stretching sheet by Adomain decomposition method (ADM). An application of modified decomposition method for MHD flow over a non-linearly stretching surface was presented by Hayat et al. [7]. Ziabakhsh et al. [8] examined the two-dimensional flow and mass transfer over a non-linearly stretching sheet immersed in a porous space. Finite element solutions for two-dimensional flow of nanofluid due to non-linearly stretching surface were computed by Bhargava et al. [9]. Shahzad et al. [10] examined the axisymmetric flow over a non-linearly stretching sheet. They computed an exact solution when the velocity of the stretching sheet was proportional to r^3. Partial slip effects on the boundary layer flow past a non-linearly permeable stretching surface have been addressed by Mukhopadhyay [11]. In another paper, Mukhopadhyay [12] analyzed the flow and heat transfer of Casson fluid due to non-linearly stretching sheet. Rashidi et al. [13] derived homotopy based analytic solutions for flow over a non-isothermal stretching plate with transpiration. Cortell [14] discussed non-linear radiation heat transfer in the two-dimensional flow due to non-linearly stretching sheet. In a recent paper, Cortell [15] discussed the flow of viscoelastic fluid over a quadratic stretching sheet under the influence of non-linear Rosseland thermal radiation.

The quest for sustainable energy generation to overcome global energy crisis has resulted in new research avenues for scientists and engineers across the world. Solar energy is considered as one of the best sources of pollution free renewable energy. The idea of using small particles to collect solar energy was first investigated by Hunt [16] in the 1970s. Researchers concluded that optimal

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Nomenclature

(x, y, z)	Cartesian coordinate system
u, v, w	velocity components along the x-, y-, z-directions
$u_w,\ v_w$	velocity of the stretching sheet along the x- and y-directions
T	local fluid temperature
T_w	wall temperature
T_∞	ambient fluid temperature
C	local nanoparticle volume fraction
C_∞	ambient nanoparticle volume fraction
a, b, n	positive constants
Pr	Prandtl number
Sc	Schmidt number
f, g	dimensionless x- and y-components of velocity
k	thermal conductivity
Nu	local Nusselt number
Nur	reduced Nusselt number
q_w	wall heat flux
D_B	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
Nt	thermophoresis parameter
$\mathrm{Re}_x,\ \mathrm{Re}_y$	local Reynolds number along the x- and y-directions
$C_{fx},\ C_{fy}$	skin friction coefficients along the x- and y-directions
Nb	Brownian motion parameter
$'$	1st order derivative with respect to η
$''$	2nd order derivative with respect to η
$'''$	3rd order derivative with respect to η

Greek symbols

τ	effective heat capacity ratio of the nanoparticle to the base fluid
ν_f	kinematic viscosity of the fluid
α	thermal diffusivity
η	similarity variable
λ	ratio of the stretching rates
μ	dynamic viscosity
ρ_f	density of the fluid
ϕ	dimensionless nanoparticle volume fraction
$\tau_{zx},\ \tau_{zy}$	wall shear stress along x- and y-direction
θ	dimensionless temperature

utilization of solar radiations is possible through the use of nanofluid based solar collectors (see Trieb and Nitsch [17], Otanicar et al. [18] and Ladjevardi et al. [19]). With these remarkable discoveries, studies in the nanofluid flows have received significant fame in research community during the last couple of decades. In fact such flows have tremendous engineering applications such as vehicle engine cooling, solar water heating, cooling of electronic chips, cooling of transformer oil, improvement in diesel engine efficiency, performance efficiency of refrigerant/air-conditioners, etc. and many others. Masuda et al. [20] explored the variations in thermal conductivity and viscosity via dispersing ultra-fine particles in the base fluids. The terminology of nanofluids was first used by Choi and Eastman [21] when they experimentally discovered an effective way of controlling heat transfer rate using nanoparticles. Buongiorno [22] developed the non-homogeneous equilibrium mathematical model of convective transport in the flow of nanofluids. In this study he concluded that the mechanisms of Brownian motion and thermophoretic diffusion are responsible for abnormal variations in thermal properties of nanofluids. Nield and Kuznetsov [23] discussed the Cheng-Minkowycz problem of natural convective boundary layer flow in a porous medium saturated by a nanofluid. In another paper, Kuznetsov and Nield [24] investigated natural convective boundary-layer flow of a nanofluid past a vertical plate. The numerical solution for two-dimensional flow of nanofluid over a linearly stretching sheet using implicit finite difference scheme has been presented by Khan and Pop [25]. Makinde and Aziz [26] extended the work of Khan and Pop [25] for convective boundary conditions. Mustafa et al. [27] computed analytic solution for stagnation-point flow of nanofluid by using homotopy analysis method (HAM). In another paper, Mustafa et al. [28] derived analytic solutions for flow of nanofluid due to convectively heated exponentially stretching sheet. Numerical and analytic solutions for flow of nanofluid past a non-isothermal exponentially stretching sheet have been computed by Mustafa et al. [29]. MHD stagnation-point flow of an electrically conducting nanofluid past a convectively heated stretching/shrinking sheet was examined by Makinde et al. [30]. Mixed convective peristaltic flow of nanofluid through a vertical planar channel with induced magnetic field has been discussed by Mustafa et al. [31]. Exact analytic solution for flow of nanofluid with slip effect is considered by Turkyilmazoglu [32]. Sheikholeslami et al. [33] analyzed natural convection flow of nanofluid in a cavity. Entropy generation in MHD flow of nanofluid by rotating disk has been examined by Rashidi et al. [34]. MHD flow and convective heat transfer of ferrofluid has been recently examined by Sheikholeslami and Ganji [35]. Kuznetsov and Nield [36] proposed a new mathematical model for nanofluids that requires nanoparticle volume fraction to be passively rather than actively controlled. A model for three-dimensional flow of nanofluid with an application to solar energy was investigated by Junaid et al. [37].

To the best of our knowledge, there is not a single article that deals with the three-dimensional flow due to non-linearly stretching sheet. Therefore present work aims to study the three-dimensional flow of nanofluid past a plane horizontal surface stretching non-linearly in two lateral directions. The model suggested by Buongiorno [22] is adopted that accounts for combined influence of Brownian motion and thermophoretic diffusion of nanoparticles. The equations are first modeled and then solved for the numerical solutions by shooting approach. In this method, the boundary conditions are considered as a multivariate function of the initial conditions at some point. It has advantage of the faster convergence and simple implementation of the methods for initial value problems such as Runge–Kutta integration method. For this reason it has been preferred for many boundary layer flow

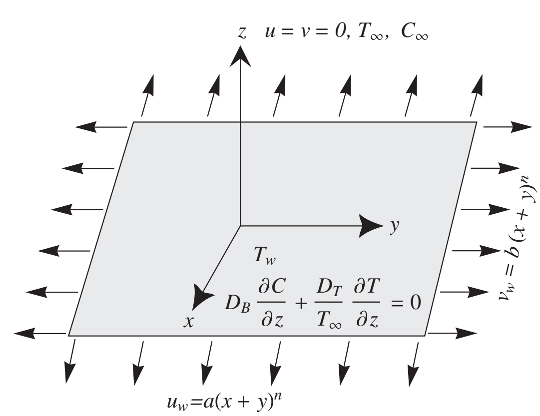

Fig. 1. Physical configuration and coordinate system.