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Thermal flux simulations by lattice Boltzmann method; investigation of high Richardson number cross flows over tandem square cylinders

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ABSTRACT

Imposing a constant non-zero heat/mass flux at a boundary is of a rather complicated type of problem with severe convergence issues especially in unsteady conditions. The present report, firstly covers the implementation of thermal lattice Boltzmann scheme and the counter-slip thermal energy density approach, to model the constant flux conditions for a typical flow of fluid over heat generating blocks. Analysis of kinetic relations shows that at certain conditions, the inflating density field could result in inconstancy of fluid heat conductivity leading to unacceptable results. Therefore to minimize such effects, density field normalization at the end of each iteration is suggested. Comparing the test case results of the current study with those of macroscopic finite volume approach in the literature, reveals the promising accuracy of the present method.

Secondly, the previously unexplored problem of steady and unsteady mixed convection over inline tandem square cylinders with a constant heat flux at the block boundary is investigated. Due to the employment of Neumann thermal boundary condition on the blocks, modified Richardson number is introduced and its consistency with the conventional definition is checked. Detailed results of the study for $10 < \text{Reynolds} < 100$, blockage ratios of 1/3, 1/4, 1/5 and 1/8 and modified Richardson numbers of 0.0, 0.5 and 1.0 are presented. The intense of the superimposed buoyance force (defined by modified Richardson number) and its impact on the onset of vortex shedding and the global quantities such as lift and drag coefficients and Nusselt number is also carefully explored.

1. Introduction

Numerous industrial applications including those of nuclear fuel rods, tubular and compact heat exchangers, electronic boards, cooling of gas turbines, oil and gas pipelines, flow dividers in polymer processing units, food treatment industries, solar heat extraction systems, chemical reactors and structural designs (buildings, masts, offshore equipment, chimneys, long-spanned bridges, etc) are the motivations behind the intensive researches on heat and fluid flow over bluff bodies. Among the wide assortment of critical parameters to investigate, a considerable effort has been made on the study of the bluff shapes (circular, square, etc), their number and arrangement (in-line, offset, etc), boundary conditions, gap to size ratio, channel confinement, flow direction (vertical or horizontal) angle of incidence and Reynolds, Richardson and Peclet (or Prandtl) Numbers. Reviews of the subject for the especial case of obstacles with circular cross sections could be found in [1–5]. Although circular and square bluff bodies are very much similar in many aspects, but the tendency of squares to fix the separation point could bring about crucial distinctions. While it is confirmed by various numerical and experimental investigations [6–8] that the separation point of rectangular shaped bodies are relatively invariable, it shows extreme sensitivity to the change of attacking angle, usually leading to an immoderate alteration of flow patterns. Igarashi [9,10] studied experimentally the influence of incidence angle and found the inclinations corresponding to extremum heat transfer rates. Biswas et al. [11] investigation on the effects of Grashof and Reynolds number on heat transfer is one of the pioneering works on mixed convection from square cylinders. They reported the perturbations of the steady wakes and the asymmetrization of the flow field as the buoyancy-induced incidents in comparison to purely forced convection conditions. In other studies, Valencia [12], Turki et al. [13], Sharma and Eswaran [14] and Dhiman et al. [15] investigated the effects of blockage ratio (*BR*) on heat and fluid flow in horizontal channels and found buildups in drag force, total Nusselt number and critical

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Nomenclature

BR	blockage ratio (D/H)
$\vec{C_i}$	lattice velocity
Cd	drag coefficient $F_D/(\frac{1}{2}\rho u_{max}D)$
Cl	lift coefficient $F_L/(\frac{1}{2}\rho u_{max}D)$
D	side of the square cylinder
e	thermal energy
e'	counter slip thermal energy
e_j	unit vector in j direction
F_b	buoyance body force
F_D	drag force
F_L	lift force
f	frequency of oscillations
$f_i, \tilde{f}_i$	particle distribution function
f_i^e	local equilibrium distribution function
g	gravitational acceleration
$\vec{G}$	volumetric body force
$g_i, \tilde{g}_i$	thermal distribution function
g_i^e	local thermal equilibrium distribution
Gr	Grashof number $\frac{g.\beta.(T-T_{ref}).D^3}{\nu^2}$
$h(x)$	local heat transfer coefficient
H	width of channel
K	thermal conductivity of the fluid
Lr	recirculation length
mGr	modified Grashof number $\frac{g.\beta.q.D^4}{k\nu^2}$
mRi	modified Richardson number $\frac{mGr}{Re^2}$
$Nu(x)$	local Nusselt number hx/K
$\overline{Nu}$	surface-averaged Nusselt number
$\langle\overline{Nu}\rangle$	period-averaged Nusselt number
P^*	dimensionless pressure $P/\rho u_{max}^2$
Pr	Prandtl number ν/α
q'	heat flux at cylinder wall
Re	Reynolds number $u_{max}D/\nu$
Re_{cr}	critical Reynolds number
R_{gas}	gas constant
Ri	Richardson number $\frac{Gr}{Re^2}$
S	spacing between blocks
St	Strouhal number fD/u_{max}
t	time
t^*	dimensionless time tu_{max}/D
T	temperature
T_w	wall temperature
T_∞	free stream temperature
T^*	dimensionless temperature $(T-T_\infty)K/qD$
$\bar{T}$	surface-averaged temperature
$u,\ v$	velocity components
$u^*,\ v^*$	dimensionless velocity components
$\vec{U}$	velocity vector $\vec{U}(u, v)$
u_{max}	maximum velocity at the inlet
$\vec{x}$	position vector
$x,\ y$	directions of Cartesian coordinates
$x^*,\ y^*$	dimensionless Cartesian coordinates $x/D,\ y/D$
Xu	upstream distance
Xd	downstream distance
Z_i	viscous dissipation
α	thermal diffusivity
β	coefficient of volume expansion
ρ	fluid density
ν	kinematic viscosity
$\mathcal{T}_f$	relaxation time
$\mathcal{T}_g$	thermal relaxation time

Reynolds number and continuous decline in recirculation length with the gradual increase of blockage ratio. The significance of Prandtl number (*Pr*) on thermal fields were studied by Dhiman et al. [15–17] and Sahu et al. [18] and they identified obvious increases in Nusselt number (*Nu*) by the rise in *Pr* in their works. The researches of various authors [13,17,19,20] on the effects of Richardson number (*Ri*) illustrates that the increase in *Ri* could lead to an upsurge in the absolute lift coefficients (due to the asymmetrization of the flow field) and Strouhal Numbers (*St*), slight increase of Nusselt numbers and expedition of the onset of vortex shedding. Vortex shedding is defined as a time-periodic oscillating wake, separated from the obstacle as a consequence of the emanated instability in the flow field (called Bénard–von Kármán instability after [21,22]). The Reynolds number at which such instabilities are emerged is called the critical Reynolds (Re_{cr}) and its magnitude depends on various factors including bluff shape, blockage ratio, Richardson number and inflow conditions. In vertical flows, effects of aiding or opposing buoyancy force could complicate the observed phenomena. The researches of Sharma and Eswaran [23] and Sharma et al. [24] into the effects of positive and negative Richardson number in vertical duct, reveal results not previously encountered in horizontal channels. Vortex shedding suppression and the degeneration of dynamic vortices into Foppl ones in particular Richardson numbers for the buoyancy aided case or on the contrary the acceleration of sheddings in opposing buoyancy circumstances are examples to name.

While all the aforementioned studies were carried out on the structures containing a single square baffle, there exist several surveys in the literature conducted on the arrangements of more than one. The great significance that inspires the study of such configurations, is the increasing complexities of both velocity and temperature fields and the observation of new phenomena stemming from the interaction of mutual shear layers. Considering the inline tandem arrangement (among the wide spectrum of arrangement possibilities) as the most prevalent type found in the literature, the new parameter of gap to size ratio is found to be of great importance.

Reports [25–30] show that the values of this factor could drastically alter the flow patterns, usually resulting in three distinguishable regimes, namely; single slender body, reattachment and co-vortex shedding. One could compare these regimes with those of synchronized, quasi-periodic and chaotic, observed in side-by-side arrangements [31,32]. Although studies of configurations containing interactions of multiple bluffs are comparatively less common, but investigations on the effects of Reynolds number [33], Richardson number [29,34,35], Prandtl [34,36] and vertical flow direction circumstances [37–39] are also available. However, considering the mentioned literature, it is well appreciated that despite the range of researches, some aspects of this subject still lack investigation. Therefore the authors concentrate on the following questions to be answered in this study:

(a) Although the influence of blockage ratio on forced convection in horizontal ducts is studied before, but to the best knowledge of the authors, the corresponding effects on mixed convection with different buoyance intensities (varying Richardson number) is not thoroughly investigated. What will be the characteristics of thermal and velocity fields for different Reynolds, blockage ratios and Richardson numbers?

(b) What are the observed phenomena for the case of three bluff bodies in a line? While most of the researches on the subject assume two inline obstacles in their simulations, the authors