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Development of a compact explicit algebraic model for the turbulent heat fluxes and its application in heated rotating flows

H. Müller [a,*], B.A. Younis [b], B. Weigand [a]

[a] Institut für Thermodynamik der Luft- und Raumfahrt, Universität Stuttgart, 70569 Stuttgart, Germany
[b] Department of Civil & Environmental Engineering, University of California, Davis, CA 95616, USA

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ABSTRACT

The paper describes the development of a compact, non-linear model for the turbulent heat fluxes, and its application to heated flow in a rotating channel. The model was formulated by expanding the tensorial functional of dependent variables, and by applying certain simplifications to obtain an algebraic and explicit model that contains direct dependence on the rotational body forces, and on the gradients of mean velocity. The model was implemented into OpenFOAM, the open-source flow solver, and its performance was assessed in stationary wall-bounded and free heated flows, and in a heated channel with spanwise rotation. Comparisons with experimental data and with results from Direct Numerical Simulations show that the new model performs better than alternative closures.

1. Introduction

The aim of the research reported herein is to improve the prediction of the turbulent heat transfer in rotating passages. This problem is of immense practical importance as it occurs in several components of power-generation and propulsion systems. In turbine blades, for example, cold air is piped through internal passages to conduct heat away from the critical parts thereby allowing for operations at elevated temperatures. The cold air is typically bled from the compressor stage of the turbine and thus its flow into these passages represents a reduction in the system's overall efficiency. There is then a clear need to reduce the flow of cold air to the minimum level that is sufficient to allow for engine operation at the design temperature. The flow and heat transfer in such cooling systems are currently predicted by using the Reynolds averaged Navier–Stokes equations (RANS) together with the energy equation. This is mainly due to the fact that the high Reynolds numbers in such applications do not allow the use of the computationally more demanding Large-Eddy Simulations (LES) or Direct Numerical Simulations (DNS) in routine design calculations. Reviews of work in this area are given for example in [1,2]. Key to the ability to predict the flow in such systems by using RANS methods is the availability of a model for the turbulent heat fluxes that can be relied upon to yield results of acceptable engineering accuracy. At present, the default model for the turbulent heat fluxes is Fourier's law in which the heat fluxes are assumed to be proportional to the temperature gradients. Absent from this model is a dependence on the mean velocity gradients and on the details of the turbulence field. Such dependence is evident from inspection of the exact equations that govern the conservation of these fluxes (Eq. 1), and its absence from Fourier's law seriously diminishes its predictive reliability in heated rotating flows. An example of where this is the case is the flow in a straight channel which is rotated about a spanwise axis. This flow has been the subject of extensive experimental and numerical studies, the latter using both RANS approaches as well as DNS (e.g. [3–5]). It was found that the effects of spanwise rotation were to reduce the turbulent mixing on the suction side relative to the equivalent stationary flow, and to enhance it on the pressure side. When rotation is sufficiently strong, the flow relaminarizes on the suction side thereby significantly reducing the heat-transfer rate from that surface leading to the formation of local hot spots. These effects are not reproducible by any gradient-transport model that has not specifically modified in some ad hoc way. In what follows, we describe the development of an explicit, algebraic model for the turbulent heat fluxes, assess this model's performance in a number of benchmark heated shear flows, and report on the outcome of its application to the prediction of flow in a heated channel with spanwise rotation.

* Corresponding author at: Institut für Thermodynamik, Universität der Bundeswehr München, Werner-Heisenberg-Weg 39, 85577 Neubiberg, Germany.
E-mail addresses: hagen.mueller@unibw.de (H. Müller), bayounis@ucdavis.edu (B.A. Younis), bernhard.weigand@itlr.uni-stuttgart.de (B. Weigand).

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Nomenclature

Latin letters

A	stress flatness parameter [–]
b_{ij}	turbulence anisotropy tensor [–]
II_b	second invariant of b_{ij} [–]
c_f	friction coefficient [–]
c_p	specific heat capacity at constant pressure [J/(kg K)]
H	channel half width [m]
k	turbulent kinetic energy [m^2/s^2]
Nu	Nusselt number [–]
p	pressure [Pa]
$p\prime$	fluctuating pressure [Pa]
P_{ij}	production of Reynolds stress [m^2/s^3]
P_k	production of turbulent kinetic energy [m^2/s^3]
Pe_t	turbulent Peclet number [–]
Pr	Prandtl number [–]
$\dot{q}_w$	wall heat flux [W/m^2]
Re	Reynolds number [–]
Re_t	turbulent Reynolds number [–]
Ro	rotation number [–]
S_{ij}	mean rate-of-strain tensor [1/s]
St	Stanton number [–]
U_b	channel bulk velocity [m/s]
u_i	fluctuating velocity component [m/s]
U_i	mean velocity component [m/s]
U_τ	friction velocity [m/s]
W_{ij}	vorticity tensor [1/s]
x_i	spatial coordinate [m]
$y+$	normalized wall distance [–]

Greek letters

α	heat transfer coefficient [W/(m^2 K)]
γ	Thermal diffusivity [m^2/s]
δ_{ij}	Kronecker delta [–]
ϵ	dissipation of turbulent kinetic energy [m^2/s^3]
ϵ_{ij}	dissipation of Reynolds stress [m^2/s^3]
ϵ_{ijk}	alternating tensor [–]
ϵ_θ	dissipation of scalar variance [K^2/s]
θ	fluctuating temperature [K]
Θ	mean temperature [K]
Θ_b	channel bulk temperature [K]
Θ_τ	friction temperature [K]
λ	thermal conductivity [W/(m K)]
ν	kinematic viscosity [m^2/s]
ρ	density [kg/m^3]
τ_{ij}	Reynolds stress tensor [m^2/s^2]
τ_w	wall friction [N/m^2]
τ_θ	scalar fluctuation time scale [s]
Ω_i	angular velocity component [1/s]

2. Model development

The starting point in the development of the model for the turbulent heat fluxes is provided by the exact equations that describe their conservation. For an incompressible fluid in an inertial frame, these equations are obtained by multiplying the equation for $(\Theta + \theta)$ by u_i and the equation for $(U_i + u_i)$ by θ, then by adding the two and time averaging the result to obtain:

$$\begin{aligned}\frac{\partial\overline{u_i\theta}}{\partial t} + U_k\frac{\partial\overline{u_i\theta}}{\partial x_k} = &\overbrace{-\overline{u_k u_i}\frac{\partial\Theta}{\partial x_k}}^{\mathbf{P_{i\theta,1}}}\overbrace{-\overline{u_k\theta}\frac{\partial U_i}{\partial x_k}}^{\mathbf{P_{i\theta,2}}}\overbrace{-2\epsilon_{ijk}\Omega_j\overline{u_k\theta}}^{\mathbf{P_{i\theta,3}}}\\ &-(\gamma+\nu)\overline{\frac{\partial\theta}{\partial x_k}\frac{\partial u_i}{\partial x_k}}\\ &-\frac{\partial}{\partial x_k}\left(\overline{u_k u_i\theta} + \frac{\overline{p'\theta}}{\rho}\delta_{ik} - \gamma\overline{u_i\frac{\partial\theta}{\partial x_k}} - \nu\overline{\theta\frac{\partial u_i}{\partial x_k}}\right)\\ &-\overline{\frac{p'}{\rho}\frac{\partial\theta}{\partial x_i}}\end{aligned} \tag{1}$$

where ν and γ are respectively the fluid kinematic viscosity and molecular diffusivity, ρ is the fluid density, and p' is the fluctuating pressure.

The terms $P_{i\theta,1}$ and $P_{i\theta,2}$ represent, respectively, the rates at which the heat fluxes are generated by the interactions between the temperature field and the Reynolds stresses, and between the heat fluxes themselves and the mean shear. The term $P_{i\theta,3}$ arises when flow is subjected to system rotation around an arbitrary axis. Experimental data and results from DNS on stationary and rotating flows indicate that the production terms represent the largest contribution to the heat-flux balances and, as such, would exert most influence in determining the magnitude of these fluxes, and their response to changes in the turbulence structure and in the mean velocity field. An algebraic model for $\overline{u_i\theta}$ must therefore explicitly depend on the Reynolds stresses and the gradients of mean velocity, as well as on the gradients of mean temperature. The parameters of the functional relationship are immediately evident from Eq. (1) which suggests:

$$\overline{u_i\theta} = f_i(\overline{u_i u_j}, S_{ij}, W_{ij}, \Theta_{,j}, \epsilon_\theta, \tau_\theta) \tag{2}$$

where τ_θ is the time scale for the turbulent scalar fluctuations. S_{ij} and W_{ij} are, respectively, the mean rate-of-strain and the vorticity tensor:

$$S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \tag{3}$$

$$W_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right) \tag{4}$$

Several alternative approaches are possible for developing an explicit model for the turbulent scalar fluxes that is consistent with Eq. (2) (e.g. [6,7]). Among these approaches is that of [8] who used the tensor representation theory developed in [9]. In this approach, a vector quantity $(\overline{u_i\theta})$ can be expressed in terms of basis vectors $\mathbb{V}_n$ (n = 1, ..., M) via a linear polynomial expansion which takes the form:

$$\overline{u_i\theta} = \sum_{n=1}^{M}\alpha_n\mathbb{V}_n \tag{5}$$

The basis vectors $\mathbb{V}_n$ are formed from the products of the symmetric (S_{ij}, τ_{ij}) and the skew-symmetric (W_{ij}) tensors, and the vector $(\Theta_{,j})$ that appears in the functional relation. The procedure is explained in detail in reference [9], and the outcome for two-dimensional flows is given by:

$$\begin{aligned}-\overline{u_i\theta} = &\,\alpha_1\Theta_{,i} + \alpha_2\tau_{ij}\Theta_{,j} + \alpha_3 S_{ij}\Theta_{,j} + \alpha_4\tau_{ik}\tau_{kj}\Theta_{,j} + \alpha_5 S_{ik}S_{kj}\Theta_{,j}\\ &+\alpha_6 W_{ij}\Theta_{,j} + \alpha_7 W_{ik}W_{kj}\Theta_{,j} + \alpha_8(S_{ik}W_{kj} + S_{jk}W_{ki})\Theta_{,j}\\ &+\alpha_9(\tau_{ik}S_{kj} - \tau_{jk}S_{ki})\Theta_{,j} + \alpha_{10}(\tau_{ik}W_{kj} + \tau_{jk}W_{ki})\Theta_{,j}\end{aligned} \tag{6}$$