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Numerical smearing, ray effect, and angular false scattering in radiation transfer computation

Brian Hunter, Zhixiong Guo *

Department of Mechanical and Aerospace Engineering, Rutgers, The State University of New Jersey, 98 Brett Road, Piscataway, NJ 08854, USA

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ABSTRACT

Solutions of the integro-differential equation of radiation transfer via numerical methods were well known to suffer from two "separate" shortcomings: (1) numerical smearing error due to spatial domain discretization, and (2) ray effect error due to angular discretization. In this study, proportionality expressions for various orders of numerical smearing errors are derived, and the inherent dependence of such errors on both spatial and angular discretization is found. Ray effect is categorized into two components: local and propagation errors; and they are not independent of spatial discretization. Using DOM solution, the individual and combined impacts of the above-mentioned numerical errors together with the recently discovered angular false scattering error are examined for various spatial and angular discretizations and medium optical properties. The dependence of numerical errors on scattering anisotropy is investigated. It is found that, for low scattering anisotropy, either numerical smearing or ray effect errors dominate, depending on optical thickness and scattering albedo. For high scattering anisotropy, however, the ray effect and angular false scattering dominate.

1. Introduction

Numerical methods have become increasingly demanded in the field of radiation heat transfer [1,2]. Processes requiring complete and accurate solutions to the equation of radiation transfer (ERT) include high-temperature combustion and material processing [3–6], fire and flame radiation [7–9], renewable solar energy [10], and biomedical therapeutic applications involving the interaction of biological tissue with light [11–14]. In the presence of radiation scattering, the ERT is an integro-differential equation, and analytic solutions are nearly impossible. Thus, to accurately determine radiation heat transfer contributions, various numerical methods, such as the discrete-ordinates method (DOM) [15–17] and Finite-Volume Method (FVM) [18–19] have been developed. Solution of the ERT using numerical methods involves the discretization of both the spatial and angular domains, which result in three types of numerical error: (1) numerical smearing, (2) ray effect, and (3) angular false scattering.

Numerical smearing is a direct result of spatial discretization practices [20–22]. Considered a counterpart of artificial numerical diffusion in computational fluid dynamics, numerical smearing is significant in multidimensional problems where spatial grid lines and radiation directions are misaligned [23], although it still persists in 1-D problems. Numerical smearing is dependent on both spatial grid resolution and chosen spatial differencing scheme. First-order schemes, such as the step (upwind), are well-known to be susceptible to large numerical smearing errors [24,25]. Higher-order spatial differencing schemes may be able to reduce numerical smearing error and increase ERT solution accuracy [22–27]. A hybrid differencing scheme, developed by Li [28] as a simple method to treat collimated irradiation, was able to reduce numerical smearing error in arbitrarily specified discrete directions. Li and coworkers additionally proposed the double rays method [29] to reduce numerical smearing error. As numerical smearing tends to mimic the behavior of scattering by artificially smoothening the intensity field [24], it is sometimes referred to as "false scattering" [20–22,25,28].

Ray effect stems from the approximation of the double integral in total solid angle 4π using a finite number of discrete radiation directions [30,31]. Lack of appropriate angular resolution can lead to physically unrealistic bumps and oscillations in the intensity field [20]. The most common remedy for treatment of ray effect is to simply increase direction number [31]; however this is computationally expensive [32]. Additional quadrature modification have been recently developed, including the Modified Discrete Ordinates Method (MDOM) [33] and the Discrete Ordinates Scheme with Infinitely Small Weights (DOS + ISW) [34]. Ray effect exists in any method where angular discretization exists, although the degree of error magnitude may differ for different solution

* Corresponding author.
E-mail address: guo@jove.rutgers.edu (Z. Guo).

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Nomenclature

g	phase-function asymmetry factor
I	radiative intensity (W/m^2 sr)
M	total number of discrete directions
$\boldsymbol{r}$	position vector
$\hat{\boldsymbol{s}}$	unit direction vector
w	discrete direction weight
Greek symbols	
σ_a, σ_s	absorption, scattering coefficients (m^{-1})
μ, η, ξ	direction cosines
Φ	scattering phase-function
ϕ, θ	radiation direction azimuthal angle and polar angle (°)
Θ	scattering angle (°)
Subscripts	
b	blackbody
i	control-volume node
N	quadrature index
Superscripts	
$'$	radiation incident direction
l, l'	radiation directions
l', l	from direction l' into direction l

methods [35] and different directional quadrature schemes [36]. Ray effect and numerical smearing have been shown to exhibit compensatory effects [20,21,23,24] in some situations, such that reduction of one error may increase another error.

It is well known that DOM angular discretization breaks down scattered energy conservation for radiation involving anisotropic scattering [37,38]. To correct it, phase-function normalization via traditional scattered energy averaging is commonly implemented [38]. Only recently was it discovered that, in addition to scattered energy, the overall phase-function asymmetry factor also becomes non-conserved after directional discretization [39–41]. Non-conservation of asymmetry factor after angular discretization results in alteration of medium scattering properties, which certainly changes ERT solution. Errors of this type are a third type of numerical error, termed as "angular false scattering" [42–43]. In order to ensure mitigation of angular false scattering in approximate methods, such as the DOM and FVM, it is critical that scattered energy and asymmetry factor are simultaneously conserved after directional discretization. A new phase-function normalization approach developed by the current authors [40] is able to achieve such concurrent conservation, leading to accurate conformity of both DOM [42,43] and FVM [44] ERT solutions with benchmark Monte Carlo predictions.

Studies on the magnitudes and impacts of numerical smearing and ray effect errors are widespread throughout the field. However, with the recent discovery of angular false scattering errors, a more thorough investigation on the appearance, magnitude, and combined effects of all three numerical errors is mandated. In this study, a detailed investigation into these three errors after DOM discretization is presented. In particular, the so-called compensatory effect between numerical smearing and ray effect is analyzed and clarified. Comparisons of DOM heat fluxes with benchmark Monte Carlo predictions are shown, in order to illustrate the appearance of each numerical error. The impact of each error is analyzed for varying medium optical properties. Additionally, the combined effects of all three errors on radiation transfer predictions are determined for varying medium properties, in order to gauge the regimes where numerical errors significantly impact DOM radiation transfer predictions.

2. Discretizations of ERT

In general vector notation, the steady-state ERT of radiation intensity I can be expressed as follows, for a gray, absorbing-emitting, and scattering medium [1,2]:

$$\hat{\boldsymbol{s}}\cdot\nabla I(\boldsymbol{r},\hat{s}) = -(\sigma_a+\sigma_s)I(\boldsymbol{r},\hat{s}) + \sigma_a I_b(\boldsymbol{r}) + \frac{\sigma_s}{4\pi}\oint_{4\pi} I(\boldsymbol{r},\hat{\boldsymbol{s}}')\Phi(\hat{\boldsymbol{s}}',\hat{\boldsymbol{s}})\boldsymbol{d}\Omega' \quad (1)$$

In the above, the spatial gradients of radiation intensity on the left-hand side are balanced on the right-hand side by three terms: (1) intensity attenuation due to both absorption and out-scattering, (2) intensity augmentation due to gray medium emission, and (3) intensity augmentation due to radiation in-scattering from any radiation direction $\hat{\boldsymbol{s}}'$.

Using the DOM, Eq. (1) can be expanded into a simultaneous set of partial differential equations in many discrete directions for a general 3-D enclosure, defined using the Cartesian coordinate system, in the following dimensionless form [2]:

$$\mu^l\frac{\partial I^l}{\partial\tau_x} + \eta^l\frac{\partial I^l}{\partial\tau_y} + \xi^l\frac{\partial I^l}{\partial\tau_z} = -I^l + (1-\omega)I_b + \frac{\omega}{4\pi}\sum_{l'=1}^{M} w^{l'}\Phi^{l'l}I^{l'},\quad l = 1,2\ldots,M \quad (2)$$

where the optical coordinates $\tau_j = (\sigma_a+\sigma_s)j$ for $j = x, y, z$, and the single-scattering albedo $\omega = \sigma_s/(\sigma_a+\sigma_s)$. Using the DOM, the continuous angular integral of radiation scattering is replaced by a sum of M total discrete directions, which are individually defined by both polar angle θ^l and azimuthal angle ϕ^l. The direction cosines $\mu^l = \sin\theta^l\cos\phi^l$, $\eta^l = \sin\theta^l\sin\phi^l$, and $\xi^l = \cos\theta^l$ correspond to the x-, y-, and z-coordinate directions, respectively. In said summation, $w^{l'}$ is the DOM quadrature weighting factor corresponding to radiation direction $\hat{\boldsymbol{s}}^{l'}$, and $\Phi^{l'l}$ is the diffuse scattering phase-function value between two arbitrary radiation directions $\hat{\boldsymbol{s}}^{l'}$ and $\hat{\boldsymbol{s}}^{l}$.

To solve Eq. (2), the spatial domain of interest is discretized into numerous control-volumes (CVs), and the spatial derivatives are approximated using control-volume (CV) differencing methods. After definition of both spatial and angular discretization schemes, as well as medium properties, the ERT can be solved using an iterative CV marching procedure [2].

2.1. Numerical smearing

Numerical smearing error arises due to spatial discretization of the computational domain [20,21]. Consider the derivative $\mu^l\frac{\partial I^l}{\partial\tau_x}$, with computational domain discretized using the CV schematic in Fig. 1, and three representative discretization schemes to approximate the derivative: the step (upwind) scheme [2], diamond (central) scheme [2], and QUICK scheme [22,23]:

$$\text{Step}:\ \mu^l\left(\frac{\partial I^l}{\partial\tau_x}\right)_i \cong \mu^l\left(\frac{I^l_i - I^l_{i-1}}{\Delta\tau_x}\right) \quad (3a)$$

$$\text{Diamond}:\ \mu^l\left(\frac{\partial I^l}{\partial\tau_x}\right)_i \cong \mu^l\left(\frac{I^i_{l+1} - I^l_{i-1}}{2\Delta\tau_x}\right) \quad (3b)$$

$$\text{QUICK}:\ \mu^l\left(\frac{\partial I^l}{\partial\tau_x}\right)_i \cong \mu^l\left(\frac{3I^i_{l+1} + 3I^l_i - 7I^l_{i-1} + I^l_{i-2}}{8\Delta\tau_x}\right) \quad (3c)$$