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A fractal analysis of permeability for fractured rocks

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ABSTRACT

Rocks with shear fractures or faults widely exist in nature such as oil/gas reservoirs, and hot dry rocks, etc. In this work, the fractal scaling law for length distribution of fractures and the relationship among the fractal dimension for fracture length distribution, fracture area porosity and the ratio of the maximum length to the minimum length of fractures are proposed. Then, a fractal model for permeability for fractured rocks is derived based on the fractal geometry theory and the famous cubic law for laminar flow in fractures. It is found that the analytical expression for permeability of fractured rocks is a function of the fractal dimension D_f for fracture area, area porosity ϕ, fracture density D, the maximum fracture length l_{max}, aperture a, the facture azimuth α and facture dip angle θ. Furthermore, a novel analytical expression for the fracture density is also proposed based on the fractal geometry theory for porous media. The validity of the fractal model is verified by comparing the model predictions with the available numerical simulations.

1. Introduction

Fractured media and rocks with shear fractures or faults widely exist in nature such as oil/gas reservoirs, and hot dry rocks, etc. Usually, the fractures are embedded in porous matrix with micro pores, which play negligible effect on the seepage characteristic, and randomly distributed fractures dominate the seepage characteristic in the media. The randomly distributed fractures are often connected to form irregular networks, and the seepage characteristic of the fracture networks has the significant influence on nuclear waste disposal [1], oil or gas exploitation [2], and geothermal energy extraction [3]. In this work, we focus our attention on the seepage characteristics of fracture networks in fractured rocks and ignore the seepage performance from micro pores in porous matrix.

Over the past four decades, many investigators studied the seepage characteristics of fracture networks/rocks and proposed several models. Snow [4] developed an analytical method for permeability of fracture networks according to parallel plane model. Kranzz et al. [5] studied the permeability of whole jointed granite and tested the parallel plane model by experiments. Koudina et al. [6] investigated the permeability of fracture networks with numerical simulation method in the three-dimensional space, they assumed that fracture network consists of polygonal shape fractures and fluid flow in each fracture meets the Darcy's law. Dreuzy et al. [7] studied the permeability of randomly fractured networks by numerical and theoretical methods in two dimensions, and they verified the validity of the model by comparing to naturally fractured networks. Klimczak et al. [8] obtained the permeability of a single fracture by parallel plate model with the fracture length and aperture satisfying power-law and verified by the numerical simulation. However, these models did not provide a quantitative relationship among the permeability of fracture networks, porosity, fracture density and microstructure parameters of fractures, such as fracture length, aperture, inclination, orientation etc.

Fractures in rocks are usually random and disorder and they have been shown to have the statistically self-similar and fractal characteristic [3,9–13]. Chang and Yortsos [10] studied the single phase fluid flow in the fractal fracture networks. Watanabe and Takahashi [3] investigated the permeability of fracture networks and heat extraction in hot dry rock by using fractal method. But, they did not propose an expression of permeability with microscopic parameters included. Jafari and Babadagli [14] obtained the permeability expression with multiple regression analysis of random fractures by the fractal geometry theory according to observed data in the well logging. In addition, their expression with several empirical constants does not include the orientation factor and microstructure parameters of fracture networks. The tree-like fractal branching networks were often considered as

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fracture networks by many investigators. Xu et al. [15,16] studied the seepage and heat transfer characteristics of fractal-like tree networks. Recently, Wang et al. [17] studied the starting pressure gradient for Bingham fluid in a special dual porosity medium with randomly distributed fractal-like tree network embedded in matrix porous media. Most recently, Zheng and Yu [18] investigated gas flow characteristics in the dual porosity medium with randomly distributed fractal-like tree networks. However, the fractal-like tree network is a kind of ideal and symmetrical network.

The purpose of the present work is to derive an analytical expression and establish a model for permeability of fracture rocks/media based on the parallel plane model (cubic law) and fractal geometry theory. The proposed permeability and the predicted fracture density will be compared with the numerical simulations.

2. Fractal characteristics for fracture networks

Many investigators [3,9–13,19–23] reported that the relationships between the length and the number of fractures exhibit the power-law, exponential and log-normal types. Torabi and Berg [19] made a comprehensive review on fault dimensions and their scaling laws, and they summarized several types of scaling laws such as the length distributions for faults and fractures in siliciclastic rocks from different scales and tectonic settings. The power-law exponents of the scaling-law between the fault length and the number of faults were found to be in the range of 1.02–2.04 and are probably influenced by factors such as stress regime, linkage of faults, sampling bias, and size of the dataset. Interested readers may consult Refs. [3,9–13,19–23] for detail.

In addition, the self-similar fractal structures of fracture networks were extensively studied [22,23], and the application in complex rock structures with the fractal technique was recently reviewed by Kruhl [24]. Velde et al. [25] and Vignes-Adler et al. [26] studied the data at several length scales with fractal method and found that the fracture networks are fractal. Barton and Zoback [27] analyzed the 2D maps of the trace length of fractures spanning ten orders, ranging from micro to large scale fractures and found that $D_f = 1.3–1.7$.

The width between two plates/walls of a fracture, i.e. the parallel plate model is used to represent the effective aperture of a fracture. Generally, the relationship between the effective aperture a and the fracture length l is given by [28,29]

$$a = \beta l^n \tag{1}$$

where β and n are the proportionality coefficient and a constant according to fracture scales, respectively. The value of $n = 1$ is important, which indicates a linear scaling law, and the fracture network is self-similarity and fractal [19,29]. Thus, in the current work the value of $n = 1$ is chosen for fractures with fractal characteristic.

Thus, Eq. (1) can be rewritten as

$$a = \beta l \tag{2}$$

Eq. (2) will be used in this work.

It is well-known that the cumulative size distribution of islands on the Earth's surface obeys the fractal scaling law [30]

$$N(S > s) \propto s^{-D/2} \tag{3a}$$

where N is the total number of island of area S greater than s, and D is the fractal dimension for the size distribution of islands. The equality in Eq. (3a) can be invoked by using s_{max} to represent the largest island on Earth to yield [31]

$$N(S > s) = \left(\frac{s_{max}}{s}\right)^{D/2} \tag{3b}$$

Eq. (3b) implies that there is only one largest island on the Earth's surface, and Majumdar and Bhushan [31] used this power-law equation to describe the contact spots on engineering surfaces, where $s_{max} = g\lambda_{max}^2$ (the maximum spot area) and $s = g\lambda^2$ (a spot area), with λ being the diameter of a spot and g being a geometry factor.

It has been shown that the length distribution of fractures satisfies the fractal scaling law [3,9–13,19,22,23,32], hence, Eq. (3b) for description of islands on the Earth's surface and spots on engineering surfaces can be extended to describe the area distribution of fractures on a fractured surface, i.e.

$$N(S \geq s) = \left(\frac{a_{max}\, l_{max}}{al}\right)^{D_f/2} \tag{3c}$$

where $a_{max}l_{max}$ represents the maximum fracture area with a_{max} and l_{max} respectively being the maximum aperture and maximum fracture length, and al refers to a fracture area with the aperture and length being a and l, respectively.

Inserting Eq. (2) into Eq. (3c), we obtain

$$N(S \geq s) = \left(\frac{\beta l_{max}^2}{\beta l^2}\right)^{D_f/2} \tag{3d}$$

Then, from Eq. (3d), the cumulative number of fractures whose length are greater than or equal to l can be expressed by the following scaling law:

$$N(L \geq l) = \left(\frac{l_{max}}{l}\right)^{D_f} \tag{4}$$

where D_f is the fractal dimension for fracture lengths, $0 < D_f < 2$ (or 3) in two (or three) dimensions; and Eq. (4) implies that there is only one fracture with the maximum length. Some investigators [3,9–13,19,32] reported that the length distribution of fractures in rocks has the self-similarity and the fractal scaling law can be described by $N \propto Cl^{-D_f}$, where C is a fitting constant, D_f is the fractal dimension for the length (l) distribution of fractures and N is the number of fractures, and this fractal scaling law is similar to Eq. (4). Eq. (4) is also the base of the box-counting method [33] for measuring the fractal dimension of fracture lengths in fracture networks, and Chelidze and Guguen [9] applied the box-counting method and found that the fractal dimension of fracture network (described by Nolen-Hoeksema and Gordon [34]) in a 2D cross section is 1.6.

Since there usually are numerous fractures in fracture networks, Eq. (4) can be considered as a continuous and differentiable function. So, differentiating Eq. (4) with respect to l, we can get the number of fractures whose lengths are in the infinitesimal rang l to $l + dl$:

$$-dN(l) = D_f l_{max}^{D_f} l^{-(D_f+1)} dl \tag{5}$$

Eq. (5) indicates that the number of fractures decreases with the increase of fracture length and $-dN(l) > 0$.

The relationship among the fractal dimension, porosity and the ratio $\lambda_{max}/\lambda_{min}$ for porous media was derived based on the assumption that pores in porous media are in the form of squares with self-similarity in sizes in the self similarity range from the minimum size λ_{min} to the maximum size λ_{max}, i.e. [35]

$$D_f = d_E + \frac{\ln \varepsilon}{\ln(\lambda_{max}/\lambda_{min})} \tag{6}$$

where ε is the effective porosity of a fractal porous medium, d_E is the Euclid dimension, and $d_E = 2$ and 3 respectively in two and three dimensions. It has been shown that Eq. (6) is valid not only for exactly self-similar fractals such as Sierpinski carpet and Sierpinski gasket but also for statistically self-similar fractal porous media.

Fractures in rocks or in fractured media are analogous to pores in porous media. Therefore, Eq. (6) can be extended to describe the