International Journal of Heat and Mass Transfer 81 (2015) 188–197

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Minimizing transient energy growth of nonlinear thermoacoustic oscillations

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ARTICLE INFO

Article history:
Received 13 July 2014
Received in revised form 7 October 2014
Accepted 12 October 2014
Available online 31 October 2014

Keywords:
Combustion instability
Premixed flame
Thermoacoustic oscillation
Transient energy growth
Heat transfer
Energy conversion

ABSTRACT

Thermoacoustic oscillations triggered by transient energy growth of flow disturbances are wanted in thermoacoustic prime movers or cooling systems. However, they are undesirable in many combustion systems, such as aero-engines, gas turbines, boilers and furnaces. In this work, minimizing transient energy growth of flow disturbances in a thermoacoustic system with Dirichlet boundary conditions is considered. For this, a simplified thermoacoustic model of a premixed laminar flame with an actuator is developed. It is formulated in state-space by expanding acoustic disturbances via Galerkin series and linearizing flame model and recasting it into the classical time-lag $\mathcal{N}-\tau$ for controllers implementation. As a linear-quadratic-regulator (LQR) is implemented, the thermoacoustic oscillations exponentially decay. However, it is associated with transient energy growth of flow disturbances. To minimize the transient energy growth, a strict dissipativity controller designed basing on LQR is then implemented. Comparison is then made between the performances of these controllers. It is found that the strict dissipativity controller achieves both exponential decay of the thermoacoustic oscillations and unity maximum transient energy growth.

1. Introduction

Heat-to-sound conversion process (i.e. Thermoacoustics) has fascinated scientists and engineers for many years now, due to its practical energy applications [1–4]. Maximizing heat-driven sound is desirable in thermoacoustic heat engines/prime movers [2–9]. However, such self-sustained pressure oscillations (also known as thermoacoustic instabilities) are unwanted in aeroengines, boilers, furnaces and gas turbines, since they can cause overheating, structural damage and costly mission failure [10–19]. Thermoacoustic instability [20–22] is generated by a feedback interaction between unsteady combustion process and acoustic disturbances present in combustion systems [23]. Unsteady combustion is an efficient monopole-like sound source and produce pressure fluctuations [24]. The pressure fluctuations [21] propagate within the combustor and partially reflect from boundaries to arrive back at the combustion zone. When unsteady heat is added in phase with the pressure fluctuations [25,26], acoustical energy increases until limit cycle oscillations [27] occur.

To minimize thermoacoustic instability, feedback control techniques [22,27] can be applied to break the coupling between unsteady heat release and pressure waves. Fig. 1 shows a schematic of a typical feedback control system (in closed-loop configuration). A classical linear controller is used to drive an actuator in response to a sensor measurement. Generally, there are two typical actuation actions. One is to vary the unsteady heat release rate by using a secondary fuel or air injector [28,29]. For example, a secondary air injector was used [30] to stabilize a lean premixed pre-vaporized swirl-involved combustor. It was found that the measured transfer functions of both the burner and the actuation systems via two methods were in good agreement.

The other actuation action is to modulate the acoustic field by using a secondary sound source such as a loudspeaker [31]. Experimental investigation of using loudspeakers to minimize combustion instabilities was performed by Campos-Delgado et al. [32] on a swirl-stabilized non-premixed combustor. The loudspeakers were driven by model-based controller such as linear quadratic Gaussian (LQG)/loop transfer recovery (LTR) and H_∞ loop-shaping. It is found that the model-based controllers resulted in better damping effect on the pressure fluctuations than the conventional phase-delay controller. Dowling and Morgans [33] used a Nyquist-based feedback controller to drive a loudspeaker to mitigate Rijke-type combustion instabilities. It was experimentally shown that about 80 dB sound pressure reduction was achieved. Detailed reviews of thermoacoustic instability and feedback control techniques were reported in previous works [27,33].

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2014.10.023
0017-9310/

Thermoacoustic systems have been shown to be non-normal [34,35], due to the presence of unsteady heat release and/or non-trivial boundary conditions. The system non-normality [34] is characterized by non-orthogonal eigenmodes. It has also been shown that in a linearly stable but non-normal thermoacoustic system, there can be significant transient energy growth of flow perturbations before their eventual decay. If the disturbances transient growth is large enough, thermoacoustic instability might be triggered by causing such disturbances to grow quickly into nonlinear limit cycles. The transient growth rate was experimentally measured on a lean-premixed gas turbine combustor [36]. However, it has been shown that the disturbances transient growth cannot be predicted by the classical linear stability theory [31], since it provides information only about the long-term evolution of the eigenmodes [20]. Neither the transient energy growth can be eliminated by using classical linear controllers [8,37].

The objective of these linear controllers is to make all the eigenmodes decay exponentially, i.e. to make the system become stable. However, when the thermoacoustic eigenmodes are non-orthogonal, controlling the dominant eigenmode alone may cause other potential eigenmode becoming unstable due to the coupling between the eigenmodes [8,38]. To explain the physics behind such phenomena, Kulkarni et al. [38] studied the non-normality and its effect on active control of thermoacoustic instability in a numerically modeled Rijke tube [21,39,40]. Multiple distributed actuators were used. The heat source–a heater was modelled by using a modified King's Law. The acoustic fluctuations were expanded using Galerkin expansion technique [35]. The basis function $\psi(x) = \mathcal{F}(j\pi x)$ representing the mode-shape was shown by Kulkarni et al. [38] to be the same as that in the absence of a mean temperature [34]. This is different from the findings of the previous works [8,35,41,42]. To eliminate transient growth, a 'transient growth controller' was recently developed and applied to a heater-involved Rijke tube with multiple actuators implemented [8,37,38]. The controller is based on the critical condition derived by Whidborne and McKernan [37]. However, the previous studies did not consider the effect of mean temperature gradient on eigenfrequencies by including it in the governing equations (basis functions), even it greatly influences the dynamics and system stability behavior [39]. Neither strict dissipativity of transient growth of flame-excited oscillations was examined or reported in previous works. Lack of such investigations partially motivated the present work.

In this work, a simplified combustion system with Dirichlet boundary conditions is considered. A premixed laminar flame is confined and a monopole-like actuator is implemented. In Section 2, the governing equations are developed in state-space. The flow perturbations are expanded by using Galerkin series. And unsteady heat release from the flame is assumed to be caused by its surface variations, which results from oncoming flow velocity fluctuations. Both nonlinear and linearized flame models are studied. In Section 3, a strict dissipativity controller designed basing on a linear-quadratic-regulator (LQR) is implemented on the model combustor to eliminate the transient growth of combustion-excited oscillations. For this, the total acoustical energy per unit cross-sectional area is chosen as a measure to characterize the flow disturbance transient growth. Energy redistribution between various eigenmodes is also studied. In Section 4, the performance of the strict dissipativity controller is evaluated and compared with that of the classical LQR one.

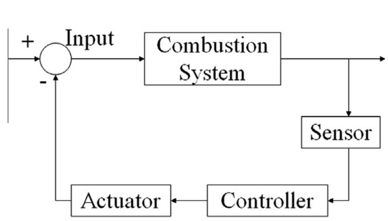

Fig. 1. Typical feedback control system scheme.

2. Description of the actuated thermoacoustic model

In the present work, we consider a simplified combustion system with a monopole-like actuator and Dirichlet boundary conditions [43,44] implemented as shown in Fig 2. A premixed laminar conical-shaped flame acts as an acoustically compact heat source. It is modelled as a thin sheet and described by using Dirac delta function in mathematics. The flow variables with a tilde are the instantaneous quantities, which consist of a mean part denoted with an overbar and a fluctuating part. The dimensional governing equations of the thermoacoustic system with an actuator comprise:

$$\bar{\rho}\frac{\partial u}{\partial t}+\frac{\partial p}{\partial x}=0 \tag{1}$$

$$\frac{1}{\bar{c}^2}\frac{\partial^2 p}{\partial t^2}+\frac{\zeta}{\bar{c}^2}\frac{\partial p}{\partial t}-\frac{\partial^2 p}{\partial x^2}=\frac{(\gamma-1)}{\bar{c}^2}\frac{\partial}{\partial t}\left(Q_s(t)\delta(x-x_f)+Q_a(t)\delta(x-x_a)\right) \tag{2}$$

where $\bar{c}$ is the sound speed, L_0 is the axial length of the combustor, γ is the ratio of specific heats, $\delta(x)$ is dirac delta function describing the localized flame or the actuator, $Q_s(t)$ and $Q_a(t)$ denote the unsteady heat release from the flame and the monopole-like actuation signal respectively.

Expanding the pressure fluctuation as Galerkin series [35,38] gives,

$$p(x,t)=\sum_{n=1}^{N}\frac{-1}{\kappa_n\omega_n}\dot{\eta}_n(t)\psi_n(x) \tag{3}$$

where

$$\kappa_n=\left[\int_0^{L_0}\psi_n^2(x)\mathrm{d}x\right]^{1/2} \tag{4}$$

N denotes the number of eigenmodes. Overdot denotes the time derivative. The basis function $\psi_n(x)$ are the eigen-solutions of the homogeneous wave equation,

$$\frac{1}{\bar{c}^2}\frac{\partial^2 p}{\partial t^2}-\frac{\partial^2 p}{\partial x^2}=0 \tag{5}$$

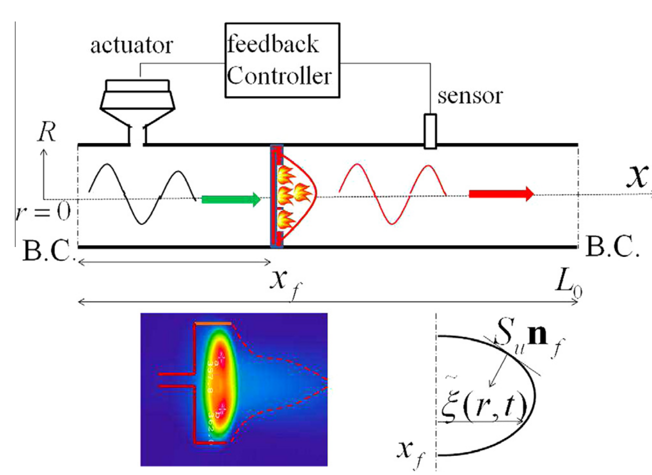

Fig. 2. Schematic of the actuated thermoacoustic model of a combustor with Dirichlet boundary conditions(BC).