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Analysis for high-order effects in thermal lagging to thermal responses in biological tissue

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ABSTRACT

For effective medical treatment processes, to understand the evolution of thermal responses in the biological tissues is important. Therefore, a non-Fourier bio-heat transfer equation, which considers the second-order effects in thermal lagging, was developed and used to investigate the variation of temperature in a laser-irradiated biological tissue. And then, thermal damage in the tissue was assessed with the rate process equation. Because the non-Fourier bio-heat transfer equation involves the mixed-derivative terms and the high-order derivatives of temperature with respect to time, there are mathematical difficulties for solving the present problem. A hybrid numerical scheme based on Laplace transform was employed. For an advance understand, the results with the non-linear effects of phase lag times are compared with the results with the linear effects. The non-linear effects of phase lag times would enhance the behavior of non-Fourier heat transfer. The diffusion-like feature of heat transfer would assist heat energy expanding and reduce thermal damage.

1. Introduction

The present work pays attentions to the further understanding of the non-Fourier effects on thermal responses within a laser-irradiated and perfused tissue. Lasers have been widely applied in the medical treatments including laser hyperthermia, interstitial thermotherapy, interstitial photocoagulation, and microsurgery. Responses of biological tissues to laser irradiation are various. The majority of those applications involve thermal effects. For effective laser treatment processes, it is important for clinical doctors to understand the evolution of temperature in the biological tissues as a result of material and heat transfer parameters that affect the laser treatment. Experiment is the most essential method for knowing the medical treatment problems. Due to the variety of tissues, development of laser technologies, and complexity of the physical and biochemical processes, it is difficult to perform a complete experiment. Therefore, theoretical investigation with mathematical models has been used as an alternative means for predicting thermal damage in laser irradiated tissue [1]. Welch [2] regarded thermal damage in a living tissue as a chemical reaction and used a chemical rate process equation to quantify it based on protein denaturation. The Arrhenius equation was accepted and used to evaluate the damage parameter by many researchers [1,3].

Thermal damage is related to the tissue temperature. Therefore, the analysis and modeling of the underlying thermal mechanisms are relatively important to further improve the thermal treatment methods. The review paper [4] presents many attempts have been made by researchers to model the complex thermal behavior of the human body. In order to solve the paradox occurred in Fourier conduction and consider the effect of micro-structural interactions in the fast transient process of heat transport, Tzou [5] proposed the dual-phase-lag (DPL) model and introduced a phase lag for temperature gradient absent in the thermal wave model. Xu et al. [6,7] presented a system discussion to the DPL effects on the biothermomechanical behavior of skin tissue. The DPL model of bio-heat transfer was, usually, developed in its first-order Taylor series expansion and accounted the linear effect of the phase lag times on thermal behavior in biological systems. This linear DPL equation has been used in various theoretical studies [8–11].

It is well known from the review paper [4] that all bioheat transfer models developed have their corresponding strengths and limitations. Although the literature [12–14] have offered the linear DPL equation of bioheat transfer the experimental support through the inverse analysis, these relevant experiments were not performed in living biological tissue. Zhang [15] and Roetzel and Xuan [16] considered that the heat transfer in living biological tissue should be the non-equilibrium heat transfer, which is much more realistic than equilibrium heat transfer assumptions. Therefore, Zhang [15] presented another feature of the DPL bio-heat

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Nomenclature

c	specific heat of tissue, J/kg K
c_b	specific heat of blood, J/kg K
f	parameter defined in Eq. (21)
g	anisotropy factor
k	thermal conductivity, W/m K
K	parameter defined in Eq. (20)
ℓ	distance between two neighboring nodes, m
n	total number of nodes
q_0	laser density, W/m^3
q_m	metabolic heat generation, W/m^3
q_r	spatial heating source, W/m^3
R_d	diffuse reflectance of light at the irradiated surface
s	Laplace transform parameter
t	time, s
T	temperature of tissue, °C
T_b	arterial temperature, °C
T_i	initial temperature of tissue, °C
T^*	new dependent variable, $T^* = T - T_b$
$\widetilde{T}^*$	Laplace transform of T^*
w_b	perfusion rate of blood, m^3/s/m^3

Greek symbols

δ	effective optical penetration depth
λ	parameter defined in Eq. (19)
μ_a	absorption coefficient
μ_s	scattering coefficient
μ_s'	reduced scattering coefficient
ρ	density, kg/m^3
τ_q	phase lag of the heat flux
τ_T	phase lag of the temperature gradient
ϕ_{in}	incident laser irradiance
Ω	damage parameter

Subscripts

i	node number
j	number of sub-space domain

transfer with the definition of separate temperatures for solid and fluid constituents of tissue. Roetzel and Xuan [16] further proposed a more refined model that splits the blood flow into arterial and venous components. As a result, Tzou and his co-workers [17,18] indicated that the second-order effects in lagging are well correlated with the number of carriers involved in non-equilibrium heat and mass transport. Xu et al. [6,7] suggested the DPL model can be further developed by taking second or higher order terms in the Taylor series expansion.

Hence, this work modifies the DPL equation of bio-heat transfer based on the second-order Taylor series expansion of phase lag times, and it is applied to simulate the thermal response in a laser irradiated tissue for the prediction of thermal damage. For convenience of statement, the present paper calls this modified DPL equation of bio-heat transfer as the non-linear DPL (NDPL) equation. This non-Fourier bio-heat transfer equation involves the mixed-derivative terms and the high-order derivatives of temperature with respect to time. The Arrhenius equation for the prediction of thermal damage is a strong non-linear function of temperature. And also, laser heating serves as a spatially-varied body heat source that heats up the tissue as scattering phenomenon is obvious and meaningful [10]. It is imaginable that there are mathematical difficulties for solving the present problem. As a result, a hybrid scheme of the Laplace transform technique and the modified discretization technique in conjunction with the hyperbolic shape functions [19,20] is extended to solve the present problem. The comparison between the results based on the DPL and NDPL equations has been made for an advance understand to the non-linear effects of thermal lagging on thermal damage in a perfused tissue. The influences of lag times, blood perfusion rate, and heating strength on the thermal damage are also discussed.

2. Problem formulation

In order to solve the paradox occurred in the classical heat flux model and consider the effect of micro-structural interactions, Tzou [5] proposed the DPL model as

$$\bar{q}(t+\tau_q) = -k\nabla T(t+\tau_T) \tag{1}$$

where T is the temperature, k the heat conductivity, q the heat flux, and t the time. τ_q means the phase lag of the heat flux and τ_T means the phase lag of the temperature gradient.

The energy conservation equation of bio-heat transfer is described as

$$\rho C\frac{\partial T}{\partial t} = -\nabla\cdot\bar{q} + w_b c_b(T_b - T) + q_m + q_r \tag{2}$$

where t is time. ρ, c, and T denote density, specific heat, and temperature of tissue. c_b and w_b are, respectively, the specific heat and perfusion rate of blood. q_m is the metabolic heat generation and q_r is the heat source for spatial heating. T_b is the arterial temperature.

Developing Eq. (1) in the second-order Taylor series expansion leads to

$$\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\bar{q} = -\left(1+\tau_T\frac{\partial}{\partial t}+\frac{\tau_T^2}{2}\frac{\partial^2}{\partial t^2}\right)k\nabla T \tag{3}$$

Substituting (3) into the energy conservation (2) leads to the second-order DPL equation of bio-heat transfer with constant physiological parameters as the following:

$$\left(1+\tau_T\frac{\partial}{\partial t}+\frac{\tau_T^2}{2}\frac{\partial^2}{\partial t^2}\right)k\nabla^2 T = \left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left[\rho c\frac{\partial T}{\partial t} - w_b c_b(T_b - T) - q_m - q_r\right] \tag{4}$$

For convenience of statement, Eq. (4) is called the non-linear DPL equation in this paper. Deleting the second-order terms of τ_q and τ_T leads to the DPL equation of bio-heat transfer.

When a broad laser beam with a uniform irradiance (ϕ_{in}) is applied normally to a finite slab of biological tissue with a thickness of L at time $t = 0^+$. Consider the spot size of the broad laser beam is much larger than the thickness of the thermally affected zone for the time period of interest, a 1-D model would be sufficient for analyzing the thermal response of the heated medium [21].

The 1-D form of (4) with constant thermal parameters is written as

$$\left(1+\tau_T\frac{\partial}{\partial t}+\frac{\tau_T^2}{2}\frac{\partial^2}{\partial t^2}\right)k\frac{\partial^2 T}{\partial x^2} = \left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left[\rho c\frac{\partial T}{\partial t} - w_b c_b(T_b - T) - q_m - q_r\right] \tag{5}$$