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Free convective flow in a vertical rectangular duct filled with porous matrix for viscosity and conductivity variable properties

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ABSTRACT

Free convection over a vertical rectangular duct filled with porous matrix with variable viscosity and variable thermal conductivity is studied in this paper. We consider the two-dimensional steady laminar flow and Brinkman–Forchheimer extended Darcy model to define the porous medium. Using the appropriate variables the basic governing equations are transformed to non-dimensional governing equations. The fluid viscosity is assumed to vary exponentially with temperature whereas the thermal conductivity is assumed to vary linearly with temperature. One of the vertical walls of the duct is cooled with constant temperature while the other wall is heated to constant but different temperature. The governing coupled nonlinear momentum and energy equations are solved numerically using finite difference method. The effect of pertinent parameters such as variable viscosity, variable thermal conductivity, Darcy number, inertial parameter, Grashof number, Brinkman number and aspect ratio on the velocity, temperature, volumetric flow rate, shear stress and heat transfer are discussed.

1. Introduction

Transport processes through porous media play important role in wide range of applications ranging from chemical catalytic reactors, thermal insulation, designing of solid-matrix heat exchangers, petroleum industries, geothermal operations, and many others. In recent years, the study of convective heat transfer and fluid flow in porous media has received great attention. Many of the earlier studies carried out by Minkowycz and Cheng [1], Cheng and Minkowycz [2], Nakayama and Koyaman [3], and Badr and Pop [4] were based on Darcy's law which states that the pressure gradient and volume-averaged velocity are directly proportional. The Darcy model is valid under the conditions of low velocities and small porosity (Hong et al. [5]). In many practical situations the porous medium which is bounded by an impermeable wall, has higher flow rates and hence reveals a non-uniform porosity distribution in the nearby wall region, hence the Darcy's law is not applicable in the present situation. For designing a real physical situation better, such as non-uniform porosity distribution and the high flow rate near the wall, it is therefore necessary to indicate the non-Darcian effects in the analysis of convective transport in a porous medium. The inertial effect is expected to be important at a higher flow rate and it can be accounted for, through the addition of a velocity squared term; well known as the Forchheimer's extension term which has been included in the momentum equation. When the heat transfer is considered in a region very close to a solid boundary, then the boundary effect may become very vital. To reveal the importance of boundary effects, the Brinkman's extension incorporating a viscous shear stress term along with the no-slip boundary condition into the momentum equation has been used. The simultaneous effects of boundary viscous resistance and the fluid inertia force, upon flow and heat transfer in a constant porosity porous medium were analyzed by Vafai and Tien [6] for forced convection and by Ranganathan and Viskantha [7] for mixed convection. Chen and Horng [8] studied the effects of non-Darcian on natural convection adjacent to a vertical cylinder embedded in thermally stratified porous medium. Rathish Kumar and Shalini [9] studied the effects of non-Darcian on natural convection heat transfer from a vertical wavy surface in a thermally stratified fluid saturated porous medium under the Forchheimer based model.

All of the preceding studies were confined to a fluid with constant properties. For the fluids, which are important in the theory of lubrication, the heat generated by the internal friction and the corresponding rise in temperature do affect the viscosity and thermal conductivity of the fluid and they can be no longer be regarded as constant. The physical properties of fluids such as viscosity and thermal conductivity may change significantly with temperature (Schlichting [10]). The temperature-dependent property problem is further complicated by the fact that with temperature the properties of different fluids behave differently. Different relations

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Nomenclature

A	aspect ratio (b/a)
a	horizontal distance
b	vertical distance
Br	Brinkman number $(\mu^3/(K\Delta T\rho^2 b^2))$
G	Grashof number $\left(g\beta_f \Delta T b^3 \rho_f^2/\mu_f^2\right)$
Da	Darcy number (κ/b^2)
BV	viscosity variation parameter $(a_1\Delta T)$
BK	conductivity variation parameter $(b_1\Delta T)$
I	inertial parameter $\left(\frac{C_F b}{\sqrt{\kappa}}\right)$
K	conductivity of the fluid
T	Temperature
T_0	reference temperature
U, V, W	velocity component
u, v, w	dimensionless velocity component
X, Y, Z	space coordinate
x, y, z	dimensionless space coordinate

Greek symbols

ρ	density
μ	viscosity
κ	permeability of the porous media
θ	dimensionless temperature

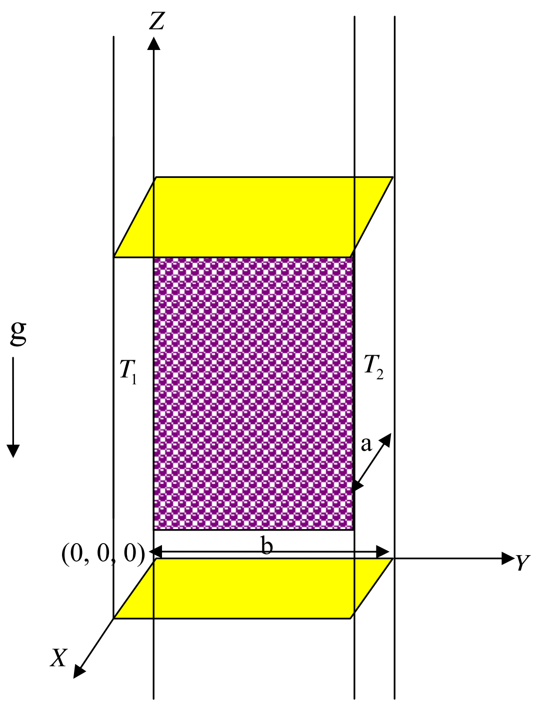

Fig. 1. Physical configuration.

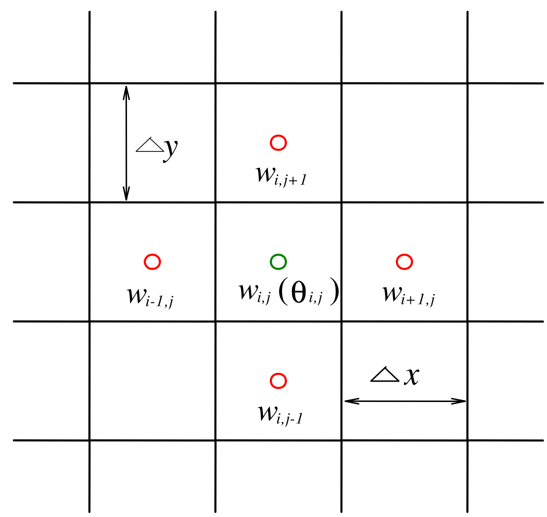

Fig. 2. Schematic plot of a uniform grid system.

between the physical properties of fluids and temperature were given by Kays and Crawford [11]. Ockendon and Ockendon [12] presented an analysis for suddenly heated or cooled channel flow of Newtonian fluids with viscosity either algebraically or exponentially dependent on temperature. Gary et al. [13] studied the effects of significant viscosity variation on convective heat transport in water-saturated porous media. Taking into account the variation of the viscosity and thermal diffusivity with temperature, Elgazery and Elazem [14] analyzed the flow of a viscous incompressible fluid along a heated vertical plate. The effects of temperature-dependent density, viscosity and thermal conductivity on the free convective steady laminar boundary layer flow by the presence of radiation for large temperature differences, was studied by Abo-Eladabah [15]. Soundalgekar et al. [16] studied flow of an incompressible viscous fluid past a continuously moving semi-infinite plate by taking into account variable viscosity and variable temperature. Elbashbeshy and Ibrahim [17] have studied the effect of variable viscosity and thermal conductivity on the heat and mass transfer in the MHD natural convective flow.

The above literature survey clearly shows that the effects of variable viscosity and variable conductivity in a vertical rectangular duct have not been studied yet. The principle aim of this paper is, therefore, to analyze the natural convection in an vertical two-dimensional rectangular duct filled with porous medium for variable physical properties. The governing non-dimensional equations are solved numerically using finite-difference method of second order accuracy.

2. Mathematical formulation

The general schematic configuration is a two-dimensional infinitely long vertical rectangular duct filled with a porous medium as shown in Fig. 1. Thermo physical properties of the fluid in the flow model assumed to vary with temperature. The Boussinesq approximation is invoked for the fluid properties to relate density changes to temperature changes, and to couple in this way the temperature field to the flow field.

The conservation equations for the porous layer are based on a non-Darcian model, incorporating the Brinkman and Forchheimer extensions. Beckermann et al. [18] have shown in their