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Effect of wall temperature in supersonic turbulent boundary layers: A numerical study

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ABSTRACT

The present work is dedicated to the numerical study of statistical characteristics of spatially-evolving supersonic turbulent boundary layers (STBL) with cooled walls. Large-Eddy Simulations (LESs) are performed to gain further insight into the role of wall temperature on the mean and fluctuating-flow properties of STBL. The velocity fluctuations, which are scaled according to the Morkovin's hypothesis, have shown acceptable agreement with available experimental and DNS results of literature. However, the van Driest transformed skin friction $C_{f_{inc}}$ lies below the incompressible theoretical curves as a function of $Re_{\theta_{inc}}$ for cold STBL, whereas compressible skin friction is found to be relatively higher for cold wall boundary layers than adiabatic boundary layers. The variation of total shear stress remains unaffected throughout the boundary layer for $0.5 \leqslant T_w/T_r \leqslant 1$. The total temperature fluctuations are non-negligible for cold cases and surface cooling changes the near-wall turbulent structures. Additionally, the streamwise velocity and temperature fluctuations for the coldest isothermal STBL case are strongly correlated compared to the anti-correlation behavior of the adiabatic STBL in the near-wall region ($y^+ \approx 9$). Furthermore, the pressure fluctuations are found to be non-negligible for cooled boundary layers and a positive correlation between pressure and density fluctuations are observed in the log-layer. These tendencies have also been verified through a detailed statistical analysis of the unsteady flow-field.

1. Introduction

Supersonic turbulent boundary layers (STBLs) with wall heat transfer occur in variety of internal as well as external flows, such as supersonic nozzles and high-speed airfoils. Variable wall properties for both low-speed as well as high-speed flows exist when large temperature gradient presents at the wall. In fact, at low-Mach number, turbulent boundary layer can be considered incompressible as long as density fluctuations remain small throughout the layer. However, when heat transfer is involved, density as well as viscosity vary with the wall distance. Hence, the turbulent structures can be greatly affected by the change of the fluid properties.

Morkovin [1] showed that '*in non-hypersonic boundary layers* [...] *the entropy mode (total-temperature) is very small for conventional rates of heat transfer*' [2], and concluded that at moderate Reynolds numbers '*the essential dynamics of these shear flows will follow the incompressible pattern*' [3]. Since then, turbulence behavior of adiabatic STBL have been widely studied and reviewed [2–4]. It is known that high-speed boundary layers can be studied using the same models as low-speed flows, as long as the variations in mean flow properties are taken into account. This assumption is certainly reasonable for supersonic flows without mass and heat transfer [5]. Nevertheless, for a zero-pressure-gradient (ZPG) compressible boundary layer, variation of density is affected by the level of heat transfer at the wall in addition to the gradient of the Mach number. Thus, a variation in wall heat transfer is expected to have a significant effect on the turbulence behavior, and the applicability of previous results learned from adiabatic flows is questionable. The present study aims to shed more light on this issue by investigating the detailed statistical behavior of a STBL subjected to strong heat transfer at the wall. The relevant question of how the surface temperature affects the compressibility of the flow and the turbulent transport properties along with the wall-scaling laws does not appear to be fully addressed in the literature so far.

Previous studies concerning heat transfer for high-speed flows dealt mainly with theoretical calculations of some mean flow properties, since measurements over those considerations (cooled walls at high-speed flows) were found to be impractical. Hopkins and

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Inouye [6] made a review of available theories [7–10] for predicting turbulent skin friction and heat transfer over flat plates for wide range of Mach numbers ($1.5 \leqslant M \leqslant 5.8$) in adiabatic as well as cooled walls ($0.3 \leqslant T_w/T_r \leqslant 1$). They suggested the use of van Driest II theory for the approximation of these quantities. Later, Cook and Richards [11] extended the Hopkins and Inouye's study with highly cooled walls ($T_w/T_r \leqslant 0.3$) at lower supersonic Mach numbers ($1.5 \leqslant M \leqslant 1.8$), and found that van Driest II theory predicts heat transfer with better certainty. However, this theory was found to be less confident for predicting skin friction for the range of considered parameters. Laderman [12] reported experimental results concerning mean flow properties of a Mach 3 turbulent boundary-layer with negligible pressure gradient, and T_w/T_r ratio ranging from 0.94 to 0.54. According to his study, the Law-of-the-Wake expression correlated to the measured mean velocity profiles and the turbulent transport properties (turbulent shear stress, mixing length and eddy viscosity) is found to be in good agreement with previous compressible adiabatic flows. With increasing heat transfer, it was observed that the total temperature–velocity profiles does not agree with the linear Crocco's relation. Following this work, Laderman and Demetriades [13] extended the turbulent transport properties results with hot-wire anemometry measurements of shear stresses of a Mach 3, zero pressure gradient, two-dimensional, flat plate compressible boundary layer at $Re_\theta \simeq 3500$, with T_w/T_r ratios of 0.94 and 0.71. The normalized shear stress distributions τ/τ_w were found to be independent of the Mach number and especially of the wall temperature, for the considered range ($0.4 \leqslant T_w/T_r \leqslant 1.0$). In terms of velocity-temperature fluctuations correlation, Gaviglio [14] and Rubesin [15] proposed two different modified formulations of the Strong Reynolds Analogy (SRA), that better accounts for the wall-heat transfer. Subsequently, Huang et al. [16] presented theoretical method for calculating skin friction and mean velocity profiles for compressible turbulent boundary layers with isothermal and adiabatic walls. Nevertheless, validation of STBL are limited by less available experimental data compared to subsonic or incompressible turbulent boundary layers. Few simulations including DNS and LES had been reported in the literature on supersonic boundary layers and channel flows [17–22]. The effects of wall-cooling/heating on turbulence structures, compressibility and turbulence scaling are poorly addressed and the literature of STBLs with heat transfer is not abundant.

Table 1 summarizes the numerical studies (DNS) available in literature for STBL over flat plate. It can be noted that compressibility effects on turbulence statistics are small up to Mach number of about 5. Success of modified SRA [23], linking between mean temperature field and streamwise velocity field and applicability of van-Driest transformation are reported by several authors. However, the total temperature fluctuations cannot be neglected, even for supersonic Mach number of 3 [24]. Decreasing the wall-temperature increases the compressibility effects [25], which influences the structure of the near wall streaks [26]. Among these studies, Duan et al. [25] performed DNS of STBLs at Mach number 5 over a wider variation of T_w/T_r ranging from 0.18 to 1. Unlike DNS, very few LES studies dealing with isothermal STBL have appeared in the literature so far. Yan et al. [27] used the monotonically integrated large-eddy simulation (MILES) technique to simulate supersonic flow (Mach number of 2.88 and 4) for adiabatic as well as isothermal case (with $T_w = 1.1\,T_r$).

In this study, a systematic approach has been undertaken to study a spatially-evolving STBL with cooled walls by means of highly resolved LESs. The influence of the wall-heat transfer on the near-wall turbulence behavior is particularly studied. From the previous overview, it can be retained that both experimental and numerical investigations do agree with the fact that, for relatively low heated/cooled walls at moderate Mach numbers, the mean velocity profiles hold for the incompressible data as long as the van-Driest transformation is applied. However, the velocity fluctuations profiles do not universally scale with the incompressible counterparts when the *standard* Morkovin scaling is used. Consequently alternative scaling laws must be applied, whether near-wall region or far-from-wall region is studied [20,21]. Also, the *original* SRA seems to fail linking velocity and temperature fluctuations, and the modified SRA relations, better accounting for the isothermal condition, are preferred [14,15,28,18]. In their recent work, Zhang et al. [29] introduced a generalized Reynolds analogy for compressible wall bounded flows. In terms of flow organization, near-wall streaks were found to be more coherent when decreasing the wall temperature [17,25] and the Morkovin's hypothesis gives good agreement for predicting those structures [26,20]. As it can be seen from Table 1, no LES/DNS studies are available with wall cooling for Mach 2 turbulent boundary layers over a flat plate. In the present paper, a spatially-evolving Mach 2 boundary layer over a flat plate for adiabatic and cooled walls are analyzed. Special attention is paid to the investigation of the influence of cooling on both mean and turbulent quantities (in particular the statistical characteristics of velocity and temperature fluctuations and their correlations). The manuscript is organized as follows: after the introduction, a brief description the governing equations as well as the numerical strategy are given in Section 2. In Section 3, the detailed study of cold-wall STBL at a fixed Mach number is presented. The influence of the wall-cooling is highlighted through a comparison with the adiabatic reference case. Finally, conclusions along with some perspectives are drawn in Section 4.

Table 1
Available DNS of STBL over flat plates. M_∞ is free stream Mach number, $\Theta = (T_w - T_\infty)/(T_r - T_\infty)$ is dimensionless temperature, and Reynolds numbers are $Re_\tau = \rho_w u_\tau \delta/\mu_w$ $Re_\theta = \rho_\delta u_\delta \theta/\mu_\delta$ $Re_{\delta_2} = \rho_\delta u_\delta \theta/\mu_w$ using standard notation.

Reference	M_∞	Θ	Re_τ	Re_θ	Re_{δ_2}
Guo and Adams [30]	3.0	1.0		3015	
	4.5	1.0		2618	
	6.0	1.0		2652	
Rai et al. [31]	2.25	1.0		6000	
Guarini [23]	2.5	1.0		1577	
Maeder [24,32]	4.5	1.0		3793	
	4.5	0.03		3953	
Pirozzoli et al. [33,34]	2.25	1.0		3953	
	2.0	1.0		4263	
Duan et al. [25]	4.97	1.0	798	1279	1538
	4.97	0.2	625	2300	1521
	4.97	0.42	522	3011	1525
	4.97	0.61	436	3819	1526
	4.97	1.0	386	4840	1537
Lagha et al. [26]	2.5–20.0	1.0	302–340		
	2.5	1.0	380		
	2.5	9.29	380		
Mayer et al. [35]	2.0	1.0			
Shahab et al. [36]	2.25	1.0		3706	913
	2.25	0.39		3798	3569
Wang et al. [37]	2.0	1.0			
Chu et al. [38]	4.9	0.38	532	3480	1817
	4.9	1.0	397	5559	1826
	4.9	1.63	353	7612	1903
Zhang et al. [29]	2.25	0.99	550		
	4.5	0.93	550		
	4.5	0.41	800		
	4.5	0.0	2100		
	6.0	0.92	550		
	3.4	0.94	377–938		

2. Governing equation and numerical methodology

The filtered compressible Navier–Stokes equations expressed in a conservative form are given by: