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Exact analysis for minimum shape of porous fins under convection and radiation heat exchange with surrounding

Balaram Kundu [a,b], Kwan-Soo Lee [a,*]

[a] School of Mechanical Engineering, Hanyang University, 17 Haengdang-dong, Sungdong-gu, Seoul 133791, Republic of Korea
[b] Department of Mechanical Engineering, Jadavpur University, Kolkata 700 032, India

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ABSTRACT

An analysis is presented for determining the minimum shape of porous fins with convection and radiation modes of heat transfer taken place on its surfaces. A variable heat transfer coefficient as a function of temperature is considered for an actual case study. An analytics based on the calculus of variation is used to establish a mathematical model. The optimum shape of porous fins for a constraint heat transfer rate is not only dependent on the thermophysical parameters of fluid flow but also a strong function with porosity. The volume of optimum profile fins monotonically increases with the porosity whatever the variation of convective heat transfer coefficient takes place. However, the requirement of solid material declines always with the porosity which may be an extremely important where gain in weight with the attachment of fins opposes always to the implementation of a final design.

1. Introduction

The use of porous fins for augmenting heat transfer is gradually attractive in heat exchanger industries for seeking more compact and more cost-effective heat exchanger manufacturing techniques. So for fin industries, this utilization has been directing ways to reduce the size and cost of fins. This demand is often justified by the high cost of the high-thermal-conductivity metals that are employed in the manufacture of finned surfaces and by the cost associated with the weight of the addition of fins.

The reduction in the size and cost of fins may only be achieved by the amount of enhancement of heat transfer carried out by the fins. There could be two different approaches for the fin optimization. Through a rigorous technique, the profile of a fin of a particular geometry (longitudinal, radial, and spine) may be obtained such that the criteria of the maximum heat transfer or the minimum volume is satisfied. In a parallel activity, the optimum dimensions of a fin of given profile shape is determined. Among these two approaches, the first approach of optimization is superior with respect to the heat transfer rate per unit volume.

Regarding the first technique, large numbers of work have been conducted to find the optimum shape of fins [1–7]. This approach is based on splitting a certain dimension of the fins in an optimal way providing that the total volume of the fin material remains fixed. Schmidt [1] was the first researcher to determine the optimum fin shape heuristically with the assumption of a constant temperature gradient satisfied through out the fin. Jany and Bejan [2] used the generalization of Schmidt's argument in the search of optimum shapes for fins, the materials of which have temperature-dependent conductivities. Natarajan and Shenoy [3] determined the optimum shape of spine for a power law spatial variation of the heat transfer coefficient. A unified analysis for determination of the optimum fin shape for longitudinal, spine, and annular fins with and without volumetric heat generation has been presented by Kundu and Das [4,5]. Under convection environment, Hanin and Campo [6] evaluated the optimum longitudinal fin shape by the relaxation of LAI assumption. In their work, a merit without consideration of LAI has been demonstrated while it has been compared with the Schmidt's optimum shape. Maday [7] obtained the optimum profile through numerical integration on Euler Lagrange condition for eliminating LAI. He suggested that an optimum fin neither has a linear temperature profile nor possesses a concave parabolic profile. Fabbri [8] proposed a genetic algorithm in order to optimize thermal performances of fin surfaces.

Using the second approach of optimization, a comprehensive review for the fin optimization has been made by Aziz [9] to find out the dimensions for a given shape by satisfying the maximization of heat transfer for a given fin volume. Yeh [10] determined

* Corresponding author. Tel.: +82 2 2220 0426; fax: +82 2 2295 9021.
E-mail addresses: bkundu@mech.net.in, bkundu123@rediffmail.com (B. Kundu), ksleehy@hanyang.ac.kr (K.-S. Lee).

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Nomenclature

c_p	specific heat of fluid at constant pressure
Da	Darcy number, K/w^2
$E(\theta, Y, L)$	dimensionless function defined in Eq. (15)
$F(\theta)$	function is dependent on θ
g	gravitational acceleration (m s^{-2})
$G(\theta)$	temperature dependent function, defined in Eqs. (48) and (62)
Gr	Grashoff number, $g\beta(T_b - T_\infty)w^3/\gamma^2$
h	temperature dependent convective heat transfer coefficient (W m^{-2} K^{-1})
h_0	convective heat transfer coefficient at the fin-base (W m^{-2} K^{-1})
k	thermal conductivity (W m^{-1} K^{-1})
K	permeability of the porous fin (m^2)
k_r	conductivity ratio, k_f/k_s
l	fin length (m)
L	dimensionless fin length, h_0/k_s
m	power factor for temperature dependent convective heat transfer coefficient
Nu	Nusselt number, $h_0 w/k_f$
Pr	Prandtl number, ν/α
$P(X)$	spatial function, defined in Eq. (21)
q	heat transfer rate through a fin (W)
R	dimensionless radiative–convective parameter, $\sigma(T_b - T_\infty)^3/h_0$
Ra	Rayleigh number, GrPr
Ra^*	modified Rayleigh number, $RaDa$
T	temperature (K)
u	fin volume (m^3)
U	dimensionless fin volume, see Eq. (14)
U^*	dimensionless material volume of a porous fin, $U(1-\phi)$
$\bar{V}_w$	average velocity of fluid (m s^{-1})
w	width of the fin (m)
x	x-coordinate acting along the length of the fin from the fin-base (m)
y	local semi-thickness of the fin (m)
X	dimensionless x-coordinate, $h_0 x/k_s$
Y	dimensionless local semi-thickness, $h_0 y/k_s$

Greek Letters

α	thermal diffusivity, $k/\rho c_p$ (m^2 s^{-1})
β	coefficient of volumetric thermal expansion (K^{-1})
β_R	Roseland extinction coefficient (L^2 kg^{-1})
δ	dimensionless conductive–convective–radiative parameter
ε	fin effectiveness
ε_m	emissivity
η	fin efficiency
λ	Lagrange multiplier
σ	Stefan–Boltzmann constant (W m^{-2} K^{-4})
ϕ	porosity
ν	kinematic viscosity (m^2 s^{-1})
ρ	density of fluid (kg m^{-3})
θ	dimensionless temperature, $(T - T_{sur})/(T_b - T_{sur})$
$\mu(X)$	spatial function, see Eq. (22)
$\psi(\theta)$	temperature dependent function, see Eq. (45)

Subscripts

b	fin-base
$cond$	conduction
eff	effective
f	fluid
L	fin-tip
s	solid
sur	surrounding

the optimum dimensions of longitudinal fins for temperature dependent heat transfer coefficient by using the asymptotic expansion of hypergeometric function. Kundu and Das [11] developed a semianalytical technique for the optimization of an eccentric annular disk fin under convection environments. The optimization study on an absorber plate fin of different geometry was investigated by Kundu [12].

The discussion above concerning the cooling fins of heat exchangers was confined to cases where the heat from the fin surface was eventually dissipated by the surrounding convective fluid. For this case with constant heat transfer coefficient, the governing equation is linear and the solution can be obtained without difficulty. However, as space technology advances, a heat exchanger may have to be designed for an environment where the heat transfer mechanism is also by radiation. Furthermore, in any space vehicle design, the overall weight of the vehicle is of utmost importance. It is, therefore, desirable to determine the fin geometry of least weight. Wilkins [13] determined the optimum shapes of fins rejecting heat by both convection and radiation. Cobble [14] investigated the shape of optimum fins subject to both convection and radiation effects. For this determination, a class of temperature function was assumed. Ahmadi and Razani [15] forwarded an approximate solution for the optimum dimensions of a convective–radiative fin.

In refrigeration and air conditioning, fin surface temperature is maintained below the surrounding temperature being cooled. If the fin surface temperature is lower than the dewpoint temperature of the surrounding air, moisture in the air is condensed and simultaneous heat and mass transfer take place on the fin surface. Kundu [16] analyzed first to determine the minimum fin shape for effectively transferring heat rate under dehumidifying conditions. A variational calculus was adopted for the analysis of three common types of fins. In his analysis, he assumed humidity ratio of saturation air as a linear function with temperature. However, the actual relationship between humidity ratio and temperature may not be a linear; it is a curve line according to the saturation curve on the psychrometric chart. This relationship can be approximated by a polynomial function [17] in order to establish an accurate model. With adopting this relationship, Kundu [18] and Kundu and Lee [19] given new techniques for determination of minimum envelop shape of fins of longitudinal and spine under fully and partially wet conditions. In an actual application, convective coefficients may vary along the fin surface for wet fins. With this consideration, Kundu and Lee [20,21] determined a least geometrical shape of fins under dehumidification. Recently, eliminating LAI, Kundu and Lee [22] investigated a new profile shape for wet fins which are always a smaller volume compared with that adopting LAI.

In the fin heat transfer literature, researchers have always been trying to choose a fin shape so that increase in weight with addition of fins will be a minimum. All the literature mentioned above are connected with the solid fin. However, if the solid geometry is converted into porous, weight of the fin definitely decreases by removing of solid material with hollow pores. This is typically called porous fins with combination of pores and matrix. The fluid