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Natural convection in a square enclosure with four circular cylinders positioned at different rectangular locations

Y.G. Park, M.Y. Ha *, J. Park

School of Mechanical Engineering, Pusan National University, Jang Jeon 2-Dong, Geum Jeong Gu, Busan 609-735, Republic of Korea

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ABSTRACT

A numerical study was carried out to investigate natural convection heat transfer induced by a temperature difference between a cold outer square enclosure and four hot inner circular cylinders placed in a rectangular array. A two-dimensional solution for natural convection in an enclosure with four cylinders was obtained using an accurate and efficient immersed boundary method. The immersed boundary method based on the finite volume method was used to handle four inner cylinders placed equal distance away from each other within an enclosure moving along the diagonal centerlines for different Rayleigh numbers in the range $10^3 \leqslant Ra \leqslant 10^6$. This study investigated the effects of the location of the cylinders in the enclosure on the heat transfer and fluid flow, when they moved along the diagonal centerlines of the enclosure. For all the cases considered in the present study, the flow and thermal fields eventually reached steady state in the range of $10^3 \leqslant Ra \leqslant 10^4$. However, for $10^5 \leqslant Ra \leqslant 10^6$, the flow and thermal fields became unsteady depending on the cylinders position. Also, the symmetry about the vertical centerline was affected by the cylinders position as well. The distribution of the instantaneous, time-averaged local and surface-averaged Nusselt number as a function of the Rayleigh number and the distance between inner cylinders was also presented.

1. Introduction

Natural convection has been vigorously studied by many scholars due to its wide range of applications from heat exchangers to nuclear safety systems and electronic equipment cooling. For several decades, number of researches has been carried out to closely examine the effect of natural convection from a single heated body placed concentrically [1–6] or eccentrically [7–11] inside a cold enclosure. However, realistic geometries used in engineering fields are more complicated than simple study involving a single body embedded either concentrically or eccentrically within a cold enclosure. Thus more information is required to fully understand the behavior of fluid flow within an enclosure with multiple bodies placed at different locations. Therefore, the effect of hot cylinders in a rectangular array at different locations in the enclosure on the heat transfer and flow characteristics has been thoroughly investigated in this study.

Previous studies which analyzed the effect of multiple hot bodies on heat transfer and flow characteristics focused on two or more bodies arranged in a linear array: horizontal or vertical or diagonal. Lacroix [12] carried out a numerical research of heat transfer due to natural convection from two vertically separated hot cylinders to a cavity cooled from above. Cavity width and top cylinder positions varied as the Rayleigh number changed from 10^4 to 10^6. The author concluded that the Nusselt number along the vertical wall-fluid interface is a function of Rayleigh number and thermal conductivity ratio.

Karimi et al. [13] carried out numerical research similar to that of Lacroix [12]. They carried out numerical simulation of natural convection from two heated horizontal cylinders in a cold square enclosure. Rayleigh numbers used varied from 10^3 to 10^7, and the dimensionless horizontal distance from 0.1 to 0.4 by increment of 0.1. The authors states that the area-averaged Nusselt number is heavily affected by the dimensionless distance between two cylinders when the Rayleigh numbers are less than 10^4, but became negligible when the Rayleigh number is between 10^4 and 10^5.

Sparrow and Niethammer [14] carried out experiments to analyze the effect of the lower cylinder on the heat transfer characteristics of the upper cylinder. The separation distance, Rayleigh number, and the temperature ratio of two cylinders varied systematically. There was no enclosure surrounding the cylinders. The upper-cylinder Nusselt number was expressed as a function of separation distance for a given temperature ratio of the cylinders and

* Corresponding author. Tel.: +82 51 510 2440; fax: +82 51 512 9835.
E-mail address: myha@pusan.ac.kr (M.Y. Ha).

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Nomenclature

f_i	momentum forcing
g	acceleration of gravity [m/s^2]
L	length of square enclosure [m]
n	normal direction to the wall
Nu	local Nusselt number
$\overline{Nu}$	surface-averaged Nusselt number
$\langle Nu \rangle$	time-averaged local Nusselt number
$\overline{\langle Nu \rangle}$	time and surface-averaged Nusselt number
P^*	pressure [Pa]
P	dimensionless pressure $(=\frac{P^* L^2}{\rho \alpha^2})$
Pr	Prandtl number (=ν/α)
r	dimensionless radius of the cylinder (=R/L)
R	radius of circular cylinder [m]
Ra	Rayleigh number $(=\frac{g\beta L^3 (T_h - T_c)}{\nu\alpha})$
t^*	time [s]
t	dimensionless time $(=\frac{t^* \alpha}{L^2})$
T	dimensional temperature [K]
T_h	hot temperature [K]
T_c	cold temperature [K]
u_i^*	velocity [m/s]
u_i	dimensionless velocity $(=\frac{u_i^* L}{\alpha})$
x_i^*	Cartesian coordinates [m]
x_i	dimensionless Cartesian coordinates $(=\frac{x_i^*}{L})$
Greek symbols	
α	thermal diffusivity [m^2/s]
β	thermal expansion coefficient [K^{-1}]
ε	dimensionless distance between the four cylinders
ε^*	dimensional distance between the four cylinders
δ_{i2}	Kronecker delta
ρ	density [kg/m^3]
ν	kinematic viscosity [m^2/s]
φ	angle from the top of the circular cylinder
θ	dimensionless temperature $(=\frac{T - T_c}{T_h - T_c})$
Superscripts/subscripts	
*	dimensional value
-	surface-averaged quantity
en	enclosure
lower left cyl	lower left cylinder
lower right cyl	lower right cylinder
upper left cyl	upper left cylinder
upper right cyl	upper right cylinder

the Rayleigh number of the upper-cylinder. The Nusselt number of the upper-cylinder was smaller than that of single cylinder when the separation distance was small, however, the Nusselt number of the upper-cylinder increased, eventually surpassing that of single cylinder, as the separation distance increased.

Saddeghipour and Asheghi [15] performed experiments to investigate the steady state free natural convection heat transfer from isothermal cylinders in vertical arrays of two to eight and Rayleigh number at 500, 600, and 700. The number of cylinders and cylinder to cylinder separation distance were varied to analyze their effect on the natural convection heat transfer behavior. Except for the bottom cylinder, the heat transfer of cylinders either reduced or improved depending on their location in the array and the geometry of the array. Authors concluded that the locations of cylinders within a specific array, number of cylinders, and geometry of the array affect the heat transfer and fluid flow characteristics in the enclosure.

Ashjaee and Yousefi [16] performed experiments to examine the laminar free convection heat transfer of isothermal and horizontal cylinders in vertical and inclined arrays. Same cylinder separation distance was used for both vertical and inclined cases. The study concluded that the free convection heat transfer from cylinder is affected by the array and position of a cylinder relative to other cylinders in array. The authors also developed heat transfer correlations for any individual cylinder in two arrays and for the entire array.

Natural convection has been studied continuously by many researchers. Despite the number of researches carried out, information about natural convection process within a cooled square enclosure containing multiple hot cylinders in specific arrays is still very scarce. As the previous studies suggest, the flow and heat transfer characteristics are heavily affected by the location of hot bodies within the enclosure and the geometry of the array of hot bodies. Therefore, the main objective of this study is to examine the effects of the locations of four hot inner cylinders in the enclosure on the heat transfer and fluid flow in the enclosure as the locations of the cylinders gradually and systematically vary with respect to the vertical and horizontal centerlines at different Rayleigh numbers. The results of the study is compared with the results from the single cylinder case in order to investigate the effects of the interaction between cylinders on the fluid flow and heat transfer in the enclosure in addition to the interaction between the hot cylinders and cold enclosure walls.

2. Numerical methodology

Fig. 1 shows the computational domain and coordinate system along with the boundary conditions considered in the present study. The system consisted of a square enclosure with side lengths L, within which four circular cylinders of radius R (=0.1L) were located and moved along the diagonal centerline in the range of $0.3 \leqslant \varepsilon \leqslant 0.7$, where $\varepsilon(=\varepsilon^*/L)$ represents the dimensionless horizontal and vertical distance from the center of a cylinder to center of another cylinder. The walls of the square enclosure were

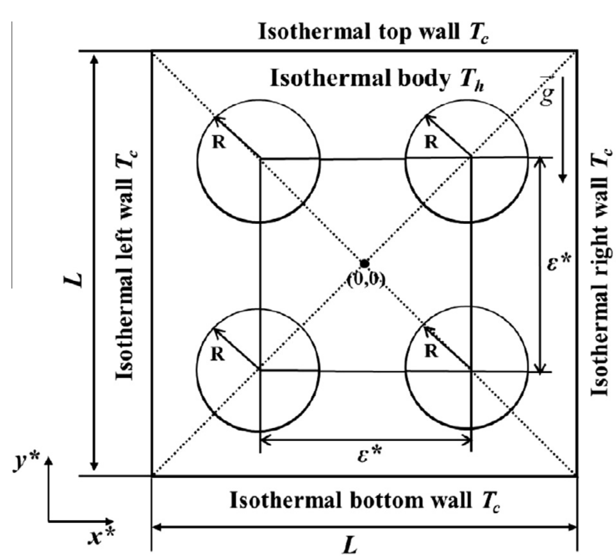

Fig. 1. Computational domain and coordinate system along with boundary conditions.