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Correlations for the double-diffusive natural convection in square enclosures induced by opposite temperature and concentration gradients

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ABSTRACT

Double-diffusive natural convection in vertical square enclosures induced by opposite horizontal temperature and concentration gradients is studied numerically. A computational code based on the SIMPLE-C algorithm for pressure–velocity coupling is used to solve the system of the conservation equations of mass, momentum, energy and species. Simulations are performed using the thermal Rayleigh number, the buoyancy ratio, the Prandtl number, and the Lewis number, as independent variables. It is found that both heat and mass transfer increase as the thermal Rayleigh number and the Prandtl number are increased, while exhibit a minimum at a value of the buoyancy ratio which increases with increasing the thermal Rayleigh number and the Lewis number. Finally, the mass transfer rate increases with the Lewis number. Conversely, the heat transfer rate is practically independent of the Lewis number as long as the buoyancy ratio is lower than the value at which the minimum heat transfer occurs, whereas it decreases significantly with the Lewis number for higher values of the buoyancy ratio. Based on the results obtained, suitable correlations are developed for the Nusselt and Sherwood numbers of the enclosure.

1. Introduction

Double-diffusive natural convection, in which buoyancy is due to simultaneously imposed temperature and concentration gradients, is of relevance to a number of natural systems, such as oceans, lakes, and magma chambers, as well as to several engineering applications, such as crystal growth, metal solidification, thin film vapor deposition, and natural gas storage, to name a few.

Over the past decades, a considerable body of research has thus been conducted in this field, both experimentally and numerically, as witnessed by an abundance of papers readily available in the literature, mainly on vertical cavities subjected to horizontal gradients, a summary of which is presented in Table 1 [1–20]. It is apparent that the square cavity ($H/L = 1$) is the most investigated setup in both cases of aiding and opposing buoyancy forces. However, although the explored range of the thermal Rayleigh number, Ra_T, and that of the absolute value of the ratio between solutal and thermal buoyancy forces, $|N|$, are in many cases adequately broad, the corresponding ranges of the Prandtl and Lewis numbers, *Pr* and *Le*, are often too narrow to take into account a meaningful number of practical situations. Moreover, it is worth pointing out that a very limited number of correlations have been proposed. In particular, for opposing flow configurations only two sets of correlations are available, both developed for air using $Le = 0.85$ [4] or $Le = 1$ [20].

Framed in this general background, a comprehensive numerical study on double-diffusive natural convection in vertical square cavities induced by opposite horizontal thermal and solutal gradients is executed with the main aim to develop suitable heat and mass transfer correlations spanning across ranges of the independent variables sufficiently wide to be of help in thermal engineering design and verification tasks. The study is performed in the hypothesis of two-dimensional laminar flow, for $Ra_T = 10^3$–10^6, $N = 0.1$–100, $Pr = 1$–1000, and $Le = 1$–1000. Notice that combinations of the lowest values of *Pr* and *Le* may correspond to dry or moist air mixed with a pollutant, whereas combinations of the highest values of *Pr* and *Le* may correspond to liquid suspensions of tiny particles, which is e.g. the case of nanofluids.

2. Mathematical formulation

A square enclosure of width *W* filled with a binary fluid is subjected to opposite horizontal thermal and solutal gradients by imposing different uniform temperatures and concentrations at

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Nomenclature

C	dimensionless concentration
c	concentration
D	mass diffusivity
$\mathbf{g}$	gravity vector
g	gravitational acceleration
Le	Lewis number
N	buoyancy ratio
Nu	Nusselt number
P	dimensionless pressure
Pr	Prandtl number
Ra	Rayleigh number
Sc	Schmidt number
Sh	Sherwood number
T	dimensionless temperature
t	temperature
U	dimensionless horizontal velocity component
$\mathbf{V}$	dimensionless velocity vector
V	dimensionless vertical velocity component
W	width of the enclosure
X, Y	dimensionless Cartesian coordinates
x, y	Cartesian coordinates

Greek symbols

α	thermal diffusivity
β	coefficient of volumetric expansion
ν	kinematic viscosity
τ	dimensionless time
ψ	dimensionless stream function

Subscripts

L	left sidewall
max	maximum
min	minimum
R	right sidewall
S	solutal
T	thermal

the vertical walls, as sketched in Fig. 1, where the reference Cartesian coordinate system (x,y) is also represented. In particular, the left sidewall, whose temperature and concentration are denoted as t_L and c_L, is the high temperature and low concentration boundary, whereas the right sidewall, whose temperature and concentration are denoted as t_R and c_R, is the low temperature and high concentration boundary, i.e., $t_L > t_R$ and $c_L < c_R$. The top and bottom walls are assumed to be perfectly insulated and impermeable. The hypothesis of zero surface emissivity is posed for the four confining walls, which physically corresponds to perfectly polished surfaces, thus implying that the present situation involves pure natural convection. The fluid flow is considered to be two-dimensional, laminar and incompressible, with constant physical properties. The buoyancy effects on momentum transfer are taken into account through the customary Boussinesq approximation. Viscous dissipation and pressure work, as well as the Soret and Dufour effects, are neglected.

Upon incorporating these hypotheses into the conservation equations of mass, momentum, energy and species, the following set of governing equations expressed in dimensionless form is obtained:

$$\nabla \cdot \mathbf{V} = 0 \tag{1}$$

$$\frac{\partial \mathbf{V}}{\partial \tau} + (\mathbf{V} \cdot \nabla)\mathbf{V} = -\nabla P + \nabla^2 \mathbf{V} - \frac{Ra_T}{Pr}(T + NC)\frac{\mathbf{g}}{g} \tag{2}$$

$$\frac{\partial T}{\partial \tau} + (\mathbf{V} \cdot \nabla)T = \frac{1}{Pr}\nabla^2 T \tag{3}$$

$$\frac{\partial C}{\partial \tau} + (\mathbf{V} \cdot \nabla)C = \frac{1}{Pr \cdot Le}\nabla^2 C \tag{4}$$

in which τ is the dimensionless time normalized by W^2/ν, $\mathbf{V}$ is the dimensionless velocity vector having horizontal and vertical components U and V normalized by ν/W, T is the dimensionless temperature excess over the uniform temperature of the right sidewall of the cavity normalized by the temperature difference $(t_L–t_R)$, C is the dimensionless concentration excess over the uniform solute concentration of the left sidewall of the cavity normalized by the concentration difference $(c_R–c_L)$, P is the dimensionless sum of the thermodynamic and hydrostatic pressures normalized by $\rho\nu^2/W^2$, $\mathbf{g}$ is the gravity vector, whereas Ra_T, N, Pr and Le are the thermal

Table 1
Summary of the studies performed on double-diffusive natural convection in rectangular enclosures.

Year	Author(s)	Method	H/L	Pr	Le	Ra_T	$\|N\|$	Aiding	Opposing	Correlation(s)
1985	Kamotani et al. [1]	Exp	0.13–0.55	7	300	$0–1.33 \times 10^7$	4–40	×	×	
1987	Trevisan and Bejan [2]	Num	1–4	0.7–7	1–40	$3.5 \times 10^5–7 \times 10^6$	0–11	×	×	
1988	Lee et al. [3]	Exp	0.2–2	4–8	50–200	$1.92 \times 10^6–2.85 \times 10^9$	2.7–72.3	×	×	
1989	Wee et al. [4]	Exp/num	7	0.7	0.85	$1.4 \times 10^5–1.4 \times 10^6$	0.005–1	×	×	Y
1990	Lee and Hyun [5]	Num	2	7	100	$2 \times 10^6–1.2 \times 10^8$	0.5–30		×	
1990	Hyun and Lee [6]	Num	2	7	100	$1.38 \times 10^6–2.76 \times 10^{10}$	0.5–10000	×		
1991	Jiang et al. [7]	Exp	0.13–0.5	7	400–425	$4 \times 10^4–2.3 \times 10^7$	2.8–102		×	
1991	Weaver and Viskanta [8]	Exp/num	1	0.44–0.85	0.58–2.03	$2.06 \times 10^4–7.35 \times 10^5$	0.55–9.42	×	×	
1991	Han and Kuehn [9]	Exp	1–4	7.7–8.9	260–330	$1.38 \times 10^6–9.2 \times 10^6$	0–23.1	×	×	
1991	Han and Kuehn [10]	Num	4	8	250	$0–3.2 \times 10^6$	0–100	×	×	
1991	Weaver and Viskanta [11,12]	Num	1	1	1	1×10^5	0–2		×	
1992	Béghein et al. [13]	Num	1	0.71	0.3–5	10^7	0–5	×		Y
1996	Bennacer and Gobin [14]	Num	1	7	1–1000	$7 \times 10^3–7 \times 10^6$	0.1–100	×		Y
1996	Gobin and Bennacer [15]	Num	1–8	7	1–1000	$7 \times 10^3–7 \times 10^6$	0.1–100	×		Y
1996	Bergman and Hyun [16]	Num	1	0.02	7500	$10^2–5 \times 10^3$	0.1–10		×	
1997	Ghorayeb and Mojtabi [17]	Num	1–15		2–151	$0–3 \times 10^4$	1		×	
1997	Costa [18]	Num	1	0.7	0.8	10^5	1–10	×	×	
2010	Sun et al. [19]	Num	1	0.71	1	2.817×10^6	1	×	×	
2012	Chen et al. [20]	Num	1	0.71	1	$10^8–10^{11}$	0.1–2		×	Y