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On the stability of internally heated natural convection due to the absorption of radiation in a laterally confined fluid layer with a horizontal throughflow

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ABSTRACT

This study focuses on the marginal stability of the natural convection flow induced by the absorption of radiation. Non-uniform internal heating associated with the absorption of radiation following Beer's law and the absorption and re-emission of residual radiation by the bottom boundary wall lead to a nonlinear base temperature profile that evolves with time. A horizontal throughflow and lateral confinement are incorporated in the analysis, and two forms of convective instability, i.e. longitudinal and transverse rolls, are investigated. It is shown that the formation of longitudinal rolls, with their axes parallel to the throughflow direction, is not affected by the presence of the throughflow. For transverse rolls, with their axes perpendicular to the throughflow direction, however, the throughflow is shown to have a stabilising effect, with the critical Rayleigh number increasing with the increasing throughflow Reynolds number. It follows that longitudinal rolls are the preferred form of instability in laterally unconfined domains for any nonzero Reynolds number and in confined domains if the Reynolds number is greater than a critical value. The value of the critical Reynolds number, which separates the preferred mode of instability, is found to rapidly reduce with increasing lateral extent and approaches zero for a normalised lateral extent greater than 2.0. The lateral confinement is also shown to have a stabilising effect for both longitudinal and transverse rolls, with the critical Rayleigh number increasing with the reducing lateral extent. It is shown that the lateral confinement only substantially affects the wave number of the rolls with their axes parallel to the confining walls but not the rolls with their axes perpendicular to the confining walls.

1. Introduction

The stability problem for the classical Rayleigh–Benard convection and a related problem incorporating a horizontal throughflow, i.e. the Rayleigh–Benard–Poiseuille flow (e.g. [1]) have been studied extensively. The effect of lateral confinement has also been investigated for the classical problem with a linear temperature stratification (e.g. [1–3]). Compared to a large body of literature available for the classical Rayleigh–Benard convection, the study on internally heated natural convection with a nonlinear temperature stratification is limited in spite of its important implications for a wide range of applications, including meteorological and oceanographical applications as mentioned in [4], limnological applications [5,6], industrial applications involving chemical vapor decomposition or the cooling of electronic equipment [7,8], the Earth's mantle convection and astrophysical plasmas [9]. Further, to the best of the authors' knowledge, none of the existing literature for internally heated natural convection has incorporated the effect of lateral confinement.

While most of the studies on internally heated natural convection focus on a spatially uniform volumetric heating, this simple model is not relevant to the problem of the convective flow induced by the absorption of radiation considered here. Such flows may be relevant to limnological applications [10,11]. For this type of flow, the volumetric heating of the fluid is non-uniform due to the absorption of radiation by a water body governed by Beer's law. Further, any residual radiation reaching the bottom bathymatry is absorbed by the bottom bathymatry and re-emitted as a boundary flux [6]. Therefore, the boundary conditions also differ from the classical Rayleigh–Benard convection, for which the fluid layer is bounded by rigid and fixed temperature walls. The relevant bottom boundary conditions for the present problem are no-slip with a fixed heat flux, while the boundary conditions at the water surface are stress-free and perfectly insulating. It is noted that the

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thermal condition at the water surface is rather complicated in the field applications and imposes a boundary forcing associated with the heat exchange between the surface and the ambient via convection, evaporation and radiation, which continuously change in response to varying meteorological conditions [12]. However, the perfectly insulating condition is selected in order to investigate the convection induced by the absorption of radiation in isolation. The coupled system involving the boundary forcing due to the surface heat exchange will be the focus of a future study.

The decay rate of the radiation intensity in a water body is controlled by the attenuation coefficient of water. While the attenuation coefficient depends on the wavelength of the radiation as well as the turbidity of the water, it is commonly approximated by a single bulk value η (e.g. [6]). The vertical (z^*) distribution of the radiation intensity is therefore given as [13]:

$$I = I_0 \exp(\eta z^*) \quad (z^* \leq 0), \tag{1}$$

where I_0 is the radiation intensity at $z^* = 0$, the water surface. A bottom boundary layer with an unstable temperature stratification forms due to the bottom boundary heat flux, whereas a stable temperature stratification, exponentially decaying from the surface, i.e. a surface layer, forms due to the direct absorption of radiation. It is known that the relative significance of the competing thermal layers depends on the value of the normalised water depth $h\eta$ [6]. For small $h\eta$, most of the incoming radiation reaches the bottom and therefore the bottom boundary layer dominates with a negligible surface layer, while for large $h\eta$ a large portion of the incoming radiation is directly absorbed with little reaching the bottom, leading to the formation of a thick surface layer. The effect of $h\eta$ on the stability of the flow in a parallelpiped cavity has been investigated via direct numerical simulation in [14].

For the possible limnological applications mentioned above, the differential heating of water columns due to the varying topography of lakes and reservoirs causes a horizontal pressure gradient which drives a horizontal throughflow. Therefore, a horizontal throughflow is incorporated in the present analysis, and consequently two forms of convective instability are considered. Longitudinal rolls have their axes parallel to the direction of the throughflow, whilst transverse rolls have their axes perpendicular to the direction of the throughflow. The linear stability of this type of flow to the formation of longitudinal rolls has been studied in [6], assuming a laterally unconfined domain.

Associated with the prescribed thermal flux conditions at the top and bottom boundaries, the base temperature solution changes with time, which again differs from the classical Rayleigh–Benard type problems. For the treatment of the time-varying base flow solution, a 'frozen time' model was employed in [6]. With the frozen time model, the base flow solution is assumed to be quasi-static, i.e. the evolution of the base flow solution does not affect its instantaneous stability. This type of assumption has been investigated widely (e.g. [15–18]), and it has been found [17] that the frozen time model is not valid for a fluid layer undergoing a step change in temperature since the critical Rayleigh number is severely underestimated. However, the flow considered here with fixed heat flux boundary conditions ensures that there are no step changes in temperature, and therefore the frozen time model is safely applied in the present study.

This study substantially extends the previous study on the natural convection induced by the absorption of radiation [6] by incorporating a horizontal throughflow and lateral confinement, and investigating both forms of convective instability, i.e. longitudinal and transverse rolls. The remainder of the paper is organised as follows: In Section 2, further details of the flow configuration and the governing equations are discussed, and the linearised perturbation equations are derived for the formations of longitudinal and transverse rolls. The numerical method based on the finite difference method is discussed in Section 3. The results of the calculations are presented and discussed in Section 4. Conclusions are then drawn in Section 5.

2. Problem formulation

The geometry of the flow domain is provided in Fig. 1. The fluid layer of the depth h is confined laterally in the y direction with the width denoted by l and is assumed to be infinite in the x direction. Longitudinal rolls have their axes parallel to the x direction, whereas transverse rolls have their axes parallel to the y direction. With the Boussinesq approximation and constant water properties assumed at ambient temperature (T_0^*), the governing equations are given in a dimensional form as:

$$\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} + w^*\frac{\partial u^*}{\partial z^*} = -\frac{1}{\rho_0}\frac{\partial p^*}{\partial x^*} + \nu\nabla^{*2}u^*, \tag{2a}$$

$$\frac{\partial v^*}{\partial t^*} + u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*} + w^*\frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho_0}\frac{\partial p^*}{\partial y^*} + \nu\nabla^{*2}v^*, \tag{2b}$$

$$\frac{\partial w^*}{\partial t^*} + u^*\frac{\partial w^*}{\partial x^*} + v^*\frac{\partial w^*}{\partial y^*} + w^*\frac{\partial w^*}{\partial z^*} = -\frac{1}{\rho_0}\frac{\partial p^*}{\partial z^*} + \nu\nabla^{*2}w^* + g\beta(T^* - T_0^*), \tag{2c}$$

$$\frac{\partial T^*}{\partial t^*} + u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} + w^*\frac{\partial T^*}{\partial z^*} = \kappa\nabla^{*2}T^* + \frac{I_0}{\rho_0 C_p}\eta\exp(\eta z^*), \tag{2d}$$

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0, \tag{2e}$$

where u^*, v^* and w^* are the velocity components in the x, y and z directions as indicated in Fig. 1; T^* is the temperature; p^* is the pressure; and $\nabla^{*2} = (\partial^2/\partial x^{*2}) + (\partial^2/\partial y^{*2}) + (\partial^2/\partial z^{*2})$. The density, kinematic viscosity, thermal diffusivity and thermal expansion coefficient are ρ_0, ν, κ and β, respectively. The gravitational acceleration is g. The internal heating source term in Eq. (2d) represents the direct absorption of radiation and decreases with depth according to Beer's law (1), where C_p is the specific heat of water. The present problem assumes that the initial condition of the flow is isothermal at T_0^* and the velocity components are zero except for the imposed throughflow. The radiation I_0 is instantaneously applied at $t^* = 0$, and maintained thereafter.

The following boundary conditions apply at the side walls ($y^* = \pm l/2$):

$$u^* = v^* = w^* = 0, \tag{3a}$$

$$\frac{\partial T^*}{\partial y^*} = 0. \tag{3b}$$

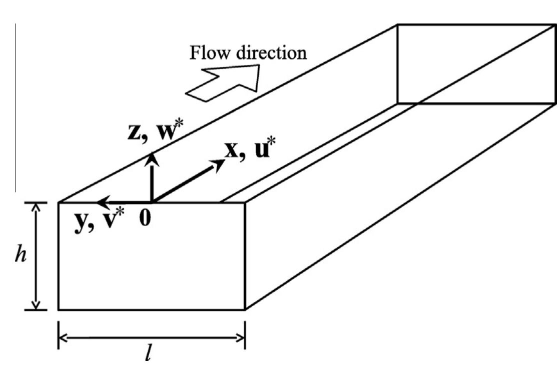

Fig. 1. Geometry of the flow domain.