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Technical Note

Theoretical study on temperature oscillation of a parallel-plate in pulsating flow condition

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ABSTRACT

Temperature distribution in a parallel-plate heater with the cooling of pulsating flow is theoretically studied. The results indicate the temperature of the heated plate fluctuates periodically as the flow pulsates. Effects of the wall thickness and its thermal properties, pulsating period and amplitude are analyzed based on the theoretical result. The fluctuation amplitude and the phase-lag of the outer surface temperature are magnified by the increase of plate thickness and the decrease of thermal diffusivity and pulsating period, and vice versa. It should be noted that the temperature difference between the outer wall and the inner wall, and the heat difference between the heat source and transported by the fluid caused by the unsteady condition should not be ignored. The results of the present study can be used to obtain a precise temperature fluctuation and the variation of the heat flux in unsteady flow condition.

1. Introduction

Pulsating flow is frequently encountered in both many engineering practices and bio-fluid aspects such as circulatory systems, heat transfer devices under ocean condition and blood flow in veins. A great many of theoretical and experimental works have been done to investigate the flow and heat transfer characteristics of the pulsating flow [1–8].

In most of the previous theoretical studies, the heat transfer characteristic is obtained based on the temperature field of the oscillatory fluid. At first, the flow field of the pulsating flow is applied using the results of classical works such as the results of Uchida [5] and Womersley [9]. Then, based on the assumption that the heat transfer to the fluid has no influence on the flow field, the solution of the temperature field is acquired by solving the energy equation [10,11]. After that, the heat transfer characteristics according to different Nusselt number definitions are discussed to analysis the effects of pulsating flow on heat transfer. The effects of the heated wall are considered by setting different boundary conditions. An analytical solution for the temperature distribution inside a solid wall subject to an acoustic standing wave was studied by Tijani [12]. In their study, a periodical heat convection boundary condition with constant heat transfer coefficient is assumed Convective heat transfer experiments in pulsating flows have also been conducted by many researchers. To measure the temperature of the heated wall, thermocouples are installed on the outer surface in order to eliminate disturbance on the flow field. The temperature of the inner surface (which fluid contact directly) is derived from steady state thermal conduction equation [13,14]. And the heat input to the fluid is obtained by assuming time-independent or constant wall temperature along the plate thickness [6,15]. Bouvier [8] performed an experimental study of heat transfer in oscillating flow inside a cylindrical tube. Heat is conducted to the inner wall surface and then transferred to the fluid. A constant heat flux density is imposed and wall temperature difference caused by the unsteady condition is considered.

From a review of the current literatures, the temperature oscillation inside a heated plate with internal heat source needs more attention. And the study of the related heat power input under the cooling of pulsating flow is also insufficient. In this paper, temperature distribution and variation in a heated parallel-plate with periodical boundary conditions are theoretically investigated. The relative temperature fluctuation amplitude, the variation of heat input to the fluid and their phase-lags are discussed.

2. Surface temperature of a heating plate with pulsating flow

Heat transfer characteristics of pulsating flow under laminar condition have been studied by many researchers [1,10]. The inner surface temperature or the average temperature of the heated wall

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can be obtained through solving the governing equation of the pulsating flow.

Under the assumption that the flow is laminar, incompressible and fully developed, the momentum and energy equations are written as the following expressions:

$$-\frac{1}{\rho_f}\frac{\partial p}{\partial z}+v_f\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t} \tag{1}$$

$$\frac{\partial T_f}{\partial t}=u\frac{\partial T_f}{\partial z}-\alpha_f\frac{\partial^2 T_f}{\partial x^2} \tag{2}$$

where ρ_f, v_f and α_f are the fluid density, kinematic viscosity and thermal diffusivity, respectively. p is the pressure, u is the velocity, and T_f is the fluid temperature. The pressure gradient imposed on the pulsating flow has been given by many researchers as the following expression:

$$\frac{\partial p}{\partial z}=\left(\frac{\partial p}{\partial z}\right)_s[1+\gamma\sin(\omega t)] \tag{3}$$

where $\left(\frac{\partial p}{\partial z}\right)_s$ is the steady part of the pressure gradient, ω is the frequency and equals to $2\pi/T_0$, in which T_0 is the oscillation period, and γ represents the oscillation amplitude of the pressure gradient. The boundary conditions of the momentum equation at the wall:

$$u|_{x=\pm h}=0 \tag{4}$$

where $2h$ is the height of the parallel-plate. By the separation variables method, the velocity can be expressed as:

$$u=u_s[1+A_u\sin(\omega t+\varphi_u)] \tag{5}$$

where u_s is the steady part of the fluid velocity and represents fully developed Poiseuille flow, A_u represents the relative amplitude of the oscillatory velocity and φ_u is the phase difference between the oscillatory velocity and pressure.

The symmetric condition at the centerline and the boundary condition at the wall of Eq. (2) are:

$$\left.\frac{\partial T}{\partial x}\right|_{x=0}=0,\quad k_f\left.\frac{\partial T}{\partial x}\right|_{x=\pm h}=q_w \tag{6}$$

where k_f is the fluid thermal conductivity and q_w is the heat input.

The temperature of the fluid takes the general form [11]

$$T=T_s[1+A_T\sin(\omega t+\varphi_T)] \tag{7}$$

where T_s is the steady part, A_T represents the relative amplitude of the oscillatory part and φ_T is the phase difference between the oscillatory velocity and pressure. The above result shows that the analytical solution of the fully developed temperature profile of pulsating flow consists of steady part and oscillatory part. The following analysis will be processed based on the assumption that the inner surface of the heated parallel-plated varies in a sinusoidal way.

3. Results

3.1. Governing equations and solutions

For a heated plate with constant volume heat generation under the cooling of pulsating flow, the governing equation, the related boundary conditions and the inertial condition are expressed as

$$\alpha\frac{\partial^2 T}{\partial x^2}+\frac{q'}{\rho c_p}=\frac{\partial T}{\partial t} \tag{8a}$$

$$T|_{x=0}=T_w \tag{8b}$$

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=\delta}=0 \tag{8c}$$

$$T|_{t=0}=T_{w1}+\frac{\delta q'}{k}x-\frac{q'}{2k}x^2 \tag{8d}$$

where k, ρ, c_p and α are the thermal conductivity, density, heat capacity and thermal diffusivity of the plate. δ and q' are the thickness of the plate and volume heat generation rate. It should be noted that two coordinate systems are used in the present study to simplify the expressions, one for flow domain and another for heated wall. The initial condition of the governing equation is the steady state solution of one-dimensional heat conduction equation and has been used by many researchers to get the inner temperature of a heated plate. T_{w1} is the inner surface temperature under steady condition. T_w is the inner surface temperature under the effect of pulsating flow and can be written as the following expression based on the previous discussions.

$$T_w=T_{w1}+T_m\sin\left(\frac{2\pi}{T_0}t\right) \tag{9}$$

where T_0 is the temperature oscillation period and T_m is the oscillatory temperature. In the above equations, second-order effects, such as the variation of physical properties are neglected for simplicity. In order to transform the above equations into a non-dimensional form, the following transformations will be applied:

$$\theta=\frac{T-T_{w1}}{q'\delta^2/k},\quad \tau=\frac{t}{T_0},\quad X=\frac{x}{\delta},\quad Fo=\frac{\alpha T_0}{\delta^2} \tag{10}$$

where Fo is the Fourier number based on the period of the temperature fluctuation. Substitute the non-dimensional parameters into Eq. (8) yields:

$$\frac{\partial^2\theta}{\partial X^2}+1=\frac{1}{Fo}\frac{\partial\theta}{\partial\tau} \tag{11a}$$

$$\theta|_{X=0}=\theta_m\sin(2\pi\tau) \tag{11b}$$

$$\left.\frac{\partial\theta}{\partial X}\right|_{X=1}=0 \tag{11c}$$

$$\theta|_{\tau=0}=X-\frac{X^2}{2} \tag{11d}$$

where θ_m is defined as

$$\theta_m=\frac{T_m}{q'\delta^2/k} \tag{12}$$

The above unsteady non-homogeneous equations are solved by separation variables method and Duhamel's theorem [16], which yields

$$\theta=X-\frac{X^2}{2}+\theta_m\zeta_{amp}\sin[2\pi\tau+\varphi]+\sum_{n=1}^{\infty}4\pi\theta_m\frac{\sin(\beta_n X)}{\beta_n}e^{-Fo\beta_n^2\tau}\times\frac{Fo\beta_n^2}{(Fo\beta_n^2)^2+(2\pi)^2} \tag{13}$$

where ζ_{amp} denotes the relative temperature fluctuation amplitude and φ is the phase difference between the local temperature and inner surface temperature T_w, which takes the form

$$\zeta_{amp}=\sqrt{(4\pi C)^2+1-8\pi C\cos\Phi} \tag{13a}$$

$$\varphi=\arctan\left(\frac{-4\pi C\cdot\sin\Phi}{1-4\pi C\cos\Phi}\right) \tag{13b}$$

where A, B, Φ and C are constants and are expressed as