International Journal of Heat and Mass Transfer 81 (2015) 465–469

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

# A new approach to transient evaporating film heating modeling based on analytical temperature profiles for internal combustion engines

Song Liye *, Zhang Weizheng, Zhang Ti'en, Qin Zhaoju

*School of Mechanical Engineering, Beijing Institute of Technology, Beijing 100081, China*

ARTICLE INFO

*Article history:*
Received 1 July 2014
Received in revised form 23 October 2014
Accepted 24 October 2014
Available online 15 November 2014

*Keywords:*
Film heating model
Analytical solution
Film evaporation
Wall flux
Engine fuel film

ABSTRACT

A new approach to modeling transient heating in evaporating fuel film was suggested, with energy governing equations of evaporating fuel film solved analytically. This approach was validated with fine numerical calculation, and only our approach could give accurate predictions compared with conventional methods in the literature. Limitations of underlying assumptions in conventional methods were revealed.



## 1. Introduction

Wall film simulation has been an issue of much interest for decades due to its significance in engineering applications. This paper concerns numerical calculation of wall film heating and evaporation in internal combustion engines. Related studies in the literature focus on the modeling of evaporation and film heating.

The calculation of fuel film evaporation in engine is usually based on vapor diffusion, by assuming fuel vapor at film surface always remains saturated. Actually, the physical process of evaporation generally involves two main phases [1]: (1) detachment of fuel molecules from the liquid surface into gas in the immediate vicinity of liquid surface, (2) diffusion of fuel vapor from surface into the ambient gas. The first phase in evaporation has been widely studied with kinetic theory and molecular dynamics. However, these studies have not been applied into industry till now due to the complexion.

Reliable evaporation modeling requires accurate prediction in film heating. Hence another key issue is film heating calculation, which is the central topic of this paper. Three results in film heating calculation are interesting, surface temperature $T_s$ and temperature gradient $\partial T/\partial x_s$, wall temperature gradient $\partial T/\partial x_w$. $T_s$ determines the saturated vapor pressure, given the pressure field; $\partial T/\partial x_s$ and $\partial T/\partial x_w$ determine the film thermal balance. In the engineering level, these three quantities influence mixture formation, combustion, emission and thermal load of wall.

Actually, film heating can be conveniently predicted in CFD code with Eulerian multiphase approach. However, that is not practical when it comes to the application in engines. In engine in-cylinder flow, length scale of fuel film at wall is nearly 1/10000 times short relative to the macro length scale. Consequently it is impossible to solve film heating equation with finite-volume-like method. Therefore, film heating has to be modelled.

Till now film heating modeling methods in the literature can be divided into two groups.

The first group of methods assume thermal mixing is infinitely rapid in film, so an average temperature is used to replace the temperature distribution. Additionally, the surface temperature is calculated by energy conservation at film surface. Hence the film temperature distribution is shown as piecewise linear shape, given wall temperature. This method was firstly used in [2], and then it was adopted by [3–5]. It is also adopted by commercial CFD codes due to its simplicity. Actually, as will be seen, the infinitely rapid mixing assumption can be accepted only when steady state will be or is reached, and it violates reality in transient state.

In the other group of methods, film temperature profiles are approximated to polynomial curves. The underlying assumption is that the real temperature distribution at any time can be fit with a single family of polynomial curves. Foucart et al. [6] developed parabolic curves for temperature profiles, and agreement was shown in the *posteriori* validation. In [7], a family of three order polynomial curves was suggested, with a shape factor defined in

* Corresponding author. Tel.: +86 1348 888 3702.
*E-mail address:* songliye@bit.edu.cn (S. Liye).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2014.10.061
0017-9310/

the polynomial coefficients. The *posteriori* agreement was roughly acceptable. In [8,9], three order polynomial curves with four independent coefficients were developed, and their prediction for $T_s$ agreed well with fine numerical solutions. Polynomial approximation method seems to have matured in engine application. Unfortunately, as will be shown in the following sections, polynomial approximation method cannot give reasonable $\partial T/\partial x_w$. $\partial T/\partial x_w$ directly controls thermal load of wall components, and it should be accurately predicted.

In this paper, a new approach to film heating modeling was presented considering deficiencies of conventional methods. Two conventional assumptions, infinitely rapid mixing and polynomial approximation, were both abandoned. The new approach was based on analytical solutions of transient energy conservation equations, together with time marching algorithm. This approach was applied to a representative practical engine case, validated with fine numerical solutions. Its advantages over conventional methods were shown, with limitations of conventional assumptions revealed.

The rest is organized as follows. Section 2 presents the mathematical formulation, followed by Section 3 the application, results and discussion. Conclusions are drawn in Section 4.

## 2. Mathematical formulation

Film heating formulation is briefly described in Section 2.1. Compatible evaporation models are introduced in Section 2.2. Time marching algorithm is presented in Section 2.3.

### 2.1. Film heating formulation

First governing equations are given, and then equations are normalized, with variables transformed. Whereafter the derived equation is solved.

#### 2.1.1. Governing equations

The diffusion in film is supposed to be dominant with the convection term ignored, and the energy flow in film is assumed one dimensional [5,11]. Multi-component nature of fuel is ignored to isolate the heating and evaporation, and its modeling can be coupled with in future works. Based on the fundamental assumptions above, the transient heat transfer equation inside the film can be written as [11]:

$$\frac{\partial T}{\partial t} = a_l \frac{\partial^2 T}{\partial x^2} \tag{1}$$

where $T = T(x,t)$ is film temperature, $t$ time, $x \in [0, \delta_0]$ distance to the wall, $\delta_0$ initial film thickness, and $a_l$ thermal diffusivity. $a_l$ is assumed constant for the analytical solution of Eq. (1). Solving equations with variable coefficients started more than 50 years ago and is still continuing [1]. In this paper, properties variation is considered in time marching algorithm, as well as boundary condition variation be. This will be discussed in Section 2.3.

Assuming that the film is heated by convection above the surface, and cooled down due to evaporation, the energy balance equation at the film surface can be written as:

$$h(T_g - T_s) = -\rho_l L\dot{\delta} + k_l \left.\frac{\partial T}{\partial x}\right|_{x=\delta_0} \tag{2}$$

where $h(t)$ is convective heat transfer coefficient, which is usually given by modified wall functions for engines [12]. $\delta$ is thickness of film, $T_g(t)$ ambient gas temperature, $\rho_l$ film density, $k_l$ film conductivity, and $L(T)$ specific heat of evaporation. $\dot{\delta} < 0$ is accounted for during the evaporation process. Eq. (2) can be considered as the boundary condition for Eq. (1) at $x = \delta_0$, complemented by $T = T_w(t)$ at $x = 0$. Initial condition is $T(x,0) = T_0(x)$ at $t = 0$.

Eq. (2) can be rearranged to:

$$T_{eff} - T_s = \frac{k_l}{h} \left.\frac{\partial T}{\partial x}\right|_{x=\delta_0} \tag{3}$$

with $T_{eff}(t) = T_g(t) + \rho_l L\dot{\delta}(t)/h(t)$. Although non-zero $\dot{\delta}$ due to evaporation is considered, $\delta$ is assumed constant in all terms except in the definition of $T_{eff}$. This assumption is acceptable when our solutions are applied in small time steps. Meanwhile, $h(t), T_g(t)$, and $T_w(t)$, together with properties, are also assumed constant. The value of $\dot{\delta}$ is given by evaporation models, which will be discussed in Section 2.2.

#### 2.1.2. Normalization and variable transformation

The governing equation with boundary and initial conditions should be normalized to make the Dirichlet boundary condition homogeneous, besides the aim for simplicity.

Normalized variables are introduced as $\Theta = (T - T_w)/(T_g - T_w)$ and $X = x/\delta_0$. Then Eq. (1) can be written as,

$$\frac{\partial \Theta}{\partial t} = \kappa \frac{\partial^2 \Theta}{\partial X^2} \tag{4}$$

where $\kappa = a_l/\delta_0^2$. Then boundary conditions become $\Theta = 0$ at $X = 0$, and $\partial\Theta/\partial X + H(t)\Theta = M(t)$ at $X = 1$, and the initial condition $\Theta_0 = \Theta_0(X,0)$ at $t = 0$, where $M(t) = \frac{(T_{eff}(t) - T_w(t))h(t)\delta_0}{(T_g(t) - T_w(t))k_l}$, and $H(t) = h(t)\delta_0/k_l$.

In a little time interval of time marching algorithm, we have $H(t) = const = H_0$, and $M(t) = \frac{(T_{eff}(t) - T_w)h\delta_0}{(T_g - T_w)k_l} = m(t)$. Then boundary condition at $X = 1$ becomes $\partial\Theta/\partial X + H_0\Theta = m(t)$.

To make boundary condition at $X = 1$ homogeneous, the new variable $W(X,t)$ is introduced satisfying the relation $\Theta(X,t) = Xm(t)/(1 + H_0) + W(X,t)$, and then Eq. (4) can be rewritten as,

$$\frac{\partial W}{\partial t} = \kappa \frac{\partial^2 W}{\partial X^2} - \frac{X}{1 + H_0}\frac{dm(t)}{dt} \tag{5}$$

with homogeneous boundary conditions $W = 0$ at $X = 0, \frac{\partial W}{\partial X} + H_0 W = 0$ at $X = 1$, and initial condition, $W_0 = \Theta_0 - \frac{X}{1+H_0}m(0)$ at $t = 0$.

#### 2.1.3. Separation of variables

Eq. (5) could be solved using separation of variables, $W = \sum_{n=1}^{\infty} c_n(t)\nu_n(X), n = 1,2,3,\ldots\infty$. Here, $\nu_n(X)$ can be derived from characteristic equation of Eq. (5) with boundary conditions, and then we will have $\nu_n(X) = \sin(\lambda_n X)$, where $\lambda$ meets the condition $\lambda\cos\lambda + H_0\sin\lambda = 0$.

$\nu_n(X), n = 1,2,3,\ldots\infty$, form a full set of eigenfunctions which are orthogonal for $X \in [0,1]$. The orthogonality of function $\nu_n(X)$ follows from relations: $\int_0^1 \nu_n(X)\nu_m(X)dX = \delta_{mn}||\nu_n||^2$, where $\delta_{mn} = 1$ if $m = n, \delta_{mn} = 0$ if $m \neq n$, and $||\nu_n||^2 = \int_0^1 \nu_n(X)\nu_n(X)dX$.

The orthogonality of $\nu_n(X)$ allows us to expand unknown functions in series: $\Theta_0(X) = \sum_{n=1}^{\infty} q_n\nu_n(X)$ and $\frac{X}{1+H_0} = \sum_{n=1}^{\infty} f_n\nu_n(X)$, where $q_n = \frac{1}{||\nu_n||^2}\int_0^1 \Theta_0(X)\nu_n(X)dX$ and $f_n = \frac{1}{||\nu_n||^2}\int_0^1 \frac{X}{1+H_0}\nu_n(X)dX$.

Substitute series of $\Theta_0(X), \frac{X}{1+H_0}$, and $W(X,t)$ into Eq. (5), and then $c_n(t)$ can be deduced. Then substituting $\nu_n(X), c_n(t)$ into $W_n(X,t)$, and with the definition of $\Theta(X,t)$, we finally obtain,

$$T(x,t) = (T_g - T_w)\left\{\frac{x/\delta_0}{1 + H_0}m(t) + \sum_{n=1}^{\infty} W_n(x/\delta_0, t)\right\} + T_w \tag{6}$$

$$W_n(x/\delta_0, t) = \left\{\exp(-\kappa\lambda_n^2 t)(q_n + f_n m(0)) + f_n\int_0^t \frac{dm(0)}{d\tau}\exp(-\kappa\lambda_n^2(t-\tau))d\tau\right\}\sin(\lambda_n x/\delta_0) \tag{7}$$