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Effect of variable viscosity on free convection in a vertical rectangular duct

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ABSTRACT

This paper considers the heat transfer in a vertical rectangular duct filled with Newtonian fluid. The viscosity of the fluid is assumed to depend on the temperature. The governing fundamental equations are approximated by a system of nonlinear ordinary differential equations and are solved numerically by using the finite difference method. The steady-state velocity and temperature contours are shown graphically. Numerical results for the skin friction, volumetric flow rate and the rate of heat transfer are obtained and reported in a tabular form for various parametric conditions to show interesting aspects of the solution. The results show that the negative values of viscosity variation parameter show intense velocity contour in the lower half region of the duct whereas positive values of viscosity variation parameter show the intense velocity contours in the upper half region of the duct. The temperature contours remain almost linear for any variations of governing parameters for all values of viscosity variation parameter.

1. Introduction

The phenomenon of natural convection in fluid-filled enclosures has received considerable attention in recent years since this phenomenon often affects the thermal performance of the systems. Buoyancy-driven flows have many applications in thermal engineering since passive cooling of electronic components by natural convection is least expensive, and most reliable method of heat rejection. Some of the applications involving enclosures are electronic packages of computer components, solar collectors and energy storage systems. Natural convection of a Boussinesq fluid in a rectangular cavity, with its two vertical side walls maintained at different constant temperatures, constitute a benchmark flow configuration. Natural convection in open ended cavities has received considerable attention by many researchers both experimentally and numerically. This attention stems from the importance of such geometry in many applications such as in electronic cooling, solar receiver systems, brake housing of an aircraft, many environmental geothermal processes and fire research. Most of the studies in this area have been aimed at two-dimensional analysis of rectangular cavities. Bejan and Kimura [1] conducted both theoretical and experimental studies to investigate the penetration of natural convection into a horizontal cavity. It was shown theoretically that the flow consists of a horizontal counter flow that penetrates the cavity over a different length, which is proportional to the cavity height and the square root of the Rayleigh number. Khanafer and Vafai [2] obtained an accurate representation of the effective boundary conditions at the aperture plane of a two-dimensional open-ended structure for a wide range of governing parameters.

Other problems involving two-dimensional natural convection in open enclosures were studied by Doria [3] for predicting fire spread in a room and by Jacobs et al. [4] and Jacobs and Mason [5] in modeling circulation above city streets and geothermal reservoirs. Experimental studies were also done by Humphrey and co-workers [6] and Sernas and Kyriakides [7] in modeling solar systems. Sernas and Kyriakides [7] studied two-dimensional laminar natural convection in an open cavity filled with air as a working fluid with cavity aspect ratio of unity. Chan and Tien [8] performed a numerical steady-state study of laminar natural convection in a two-dimensional square open cavity with a heated vertical wall and two insulated horizontal walls for Rayleigh numbers ranging from 10^3 to 10^9. Calculations were made in an external computational domain beyond the aperture plane for a Prandtl number of 1. The obtained heat transfer results were found to approach those of natural convection over a vertical isothermal flat plate. Prasad and Kulacki [9] analyzed the natural convection in a rectangular

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Nomenclature

A	aspect ratio (b/a)
a	horizontal distance
b	vertical distance
c	empirical constant
Br	Brinkman number $(\mu^3/(K\Delta T\rho^2 b^2))$
BV	variable viscosity parameter $(c\Delta T)$
Gr	Grash of number $(\vec{g}\beta\Delta T b^3\rho_0^2/\mu_0^2)$
K	conductivity of the fluid
T	temperature
T_0	reference temperature
U, V, W	velocity component
u, v, w	dimensionless velocity component
X, Y, Z	space coordinate
x, y, z	dimensionless space coordinate

Greek symbols

ρ	density
μ	viscosity
θ	dimensionless temperature

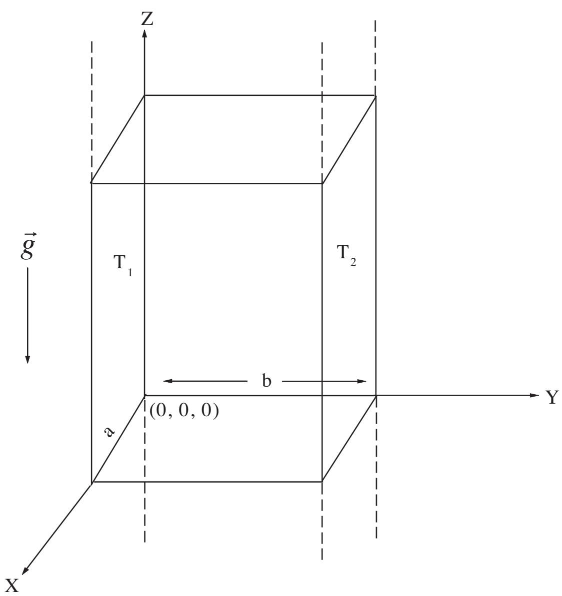

Fig. 1. Physical configuration.

Table 1
Values of volumetric flow rate skin friction and rate of heat transfer for $Gr = 10$, $Br = 2.0$, $A = 1$ for constant viscosity ($BV = 0$).

	Umavathi and Chamkha [31]	Present values
Q	4.982E-3	4.982E-00 3
$\frac{dw}{dy}\big\vert_{y=0}$	−0.307491	−0.307491
$\frac{dw}{dy}\big\vert_{y=1}$	−0.332994	−0.332994
$\frac{dw}{dx}\big\vert_{x=0}$	1.724E-2	1.724E-2
$\frac{dw}{dx}\big\vert_{x=1}$	−1.724E-2	−1.724E-2
$\frac{d\theta}{dy}\big\vert_{y=0}$	0.545424263791710	0.545424263791710
$\frac{d\theta}{dy}\big\vert_{y=1}$	0.447848031316070	0.447848031316070

cavity and the effects of aspect ratio on heat transfer and flow structure. They found that the flow patterns were observed to be quite different from those in the case of a cavity with both vertical walls at constant temperatures. Specifically, symmetry in the flow field was absent and any increase in applied heat flux was not accompanied by linearly proportional increase in the temperature on the heated wall. Also, for low Prandtl number, the heat transfer rate based upon the mean temperature difference was higher as compared to experimental results for the isothermal case. Nithirasu et al. [10] studied the buoyancy driven flow in a non-Darcian fluid saturated porous enclosure subjected to uniform heat flux using Galerkin finite element method. They concluded that different heat transfer modes were obtained for Darcy and non-Darcy flow regimes. The Nusselt number values between different Darcy numbers was found to be small. Basak et al. [11] studied natural convection in a square cavity filled with a porous medium. They found that the conduction dominant heat transfer modes for $Ra \leqslant 7 \times 10^4$ during uniform heating of bottom wall whereas the conduction dominant heat transfer was observed for $Ra \leqslant 3 \times 10^5$ for non-uniform heating (Ra is the Rayleigh number) corresponding to Darcy number equal to 10^{-4}. Umavathi et al. [12] analyzed the magnetohydrodynamic free convection flow in a vertical rectangular duct for laminar and fully developed regime taking into consideration the effects of Ohmic heating and viscous dissipation. They found that as the Hartmann number, buoyancy parameter and aspect ratio increase, the upward and downward flow rates were increased for open circuit but decrease for short circuit.

In all the above studies, throughout the flow regime, the viscosity of the fluid was assumed to be uniform. However, this physical property may change significantly with temperature and which is well known. Accordingly Garry et al. [13] and Mehta and Sood [14] have concluded that compared to the constant viscosity case, the flow characteristics change substantially when this effect is included. Kafoussius and Williams [15] and Kafoussius and Rees [16] have used the local non similarity method to investigate the effect of the temperature dependent viscosity on the mixed convection flow past a vertical flat plate in the region near the leading edge. From all these studies, they came to a conclusion that the viscosity of the fluid is sensitive to temperature variations, and the effect of temperature-dependent viscosity has to be taken into consideration; otherwise considerable errors may occur in the characteristics of the heat transfer process. Hossain et al. [17] have investigated the natural convection flow from a vertical wavy surface and Hossain and Munir [18] investigated the mixed convective flow from a vertical flat plate. These two authors treated the viscosity of the fluid to be inversely proportional to a linear function of temperature. It is worth noting that viscosity of most fluids encountered in the above mentioned applications is strongly dependent on temperature. For example, the fluidity $(1/\mu)$ of Poly-Alpha-Olefin, PAO, a synthetic lubricant that is used in cooling electronics in radar equipment, changes almost linearly with the temperature [19]. Similarly, the viscosity of glycerin has a three-fold decrease in magnitude for a 10 °C rise in temperature, while the viscosity of water decreases by 240% when the temperature increases from 10 to 50 °C. Attia [20] considered unsteady hydromagnetic Couette flow of dusty fluid with temperature dependent viscosity and thermal conductivity under exponentially decaying pressure gradient and solved the governing equations numerically using finite difference method. Recently, Das [21] investigated the influence of thermophoresis and chemical reaction on magnetohydrodynamic micropolar fluid flow with variable