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Thermal convective instability in an Oldroyd-B nanofluid saturated porous layer

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The onset of convective instability in a layer of porous medium saturated by the Oldroyd-B viscoelastic nanofluid heated from below is investigated by incorporating the effects of Brownian diffusion and thermophoresis. The flux of volume fraction of nanoparticles is taken to be zero on the boundaries. The resulting eigenvalue problem is solved numerically using the Galerkin method. The onset of convective instability is oscillatory only if the strain retardation parameter is less than the stress relaxation parameter and also when the strain retardation parameter does not exceed a threshold value which in turn depends on other physical parameters. The oscillatory onset is delayed with increasing strain retardation parameter, while an opposite trend is noticed with increasing stress relaxation parameter. The effect of increasing modified diffusivity ratio, concentration Darcy–Rayleigh number, modified particle density increment and Lewis number is to hasten the onset of stationary and oscillatory convection and also to decrease the ranges of the strain retardation parameter within which oscillatory convection is preferred.

1. Introduction

The term ‘nanofluid’ was first coined by Choi [1] and such a fluid is envisioned to describe a fluid in which nanometer-sized particles (10–100 nm) are stably suspended in conventional heat transfer basic fluids. Materials commonly used for nanoparticles include oxides such as alumina, silica, titania and copper oxide, and metals such as copper and gold. Carbon nanotubes and diamond nanoparticles have also been used to realize nanofluids. Popular base fluids include water, oil and organic fluids such as ethanol, propylene glycol and ethylene glycol. Relative to the base fluid, it has been observed consistently by many researchers that the nanofluids have abnormal thermal conductivity, viscosity and single-phase convective heat transfer coefficient. These fluids are considered to offer important advantages over conventional heat transfer fluids. The recent review articles by Kakac and Pramuanjaroenkij [2], Yu and Xie [3], Goharshadi et al. [4], Mahbubul et al. [5] have covered the latest developments in this field in detail.

In recent years, buoyancy driven convection in nanofluids has attracted researchers and has been a subject of intense current interest. Tzou [6,7] studied buoyancy driven convection in a horizontal nanofluid layer heated from below on the basis of the transport equations developed by Buongiorno [8], while Kim et al. [9] treated the Bénard problem for nanofluids in a different context. Its counterpart in a porous medium, the Darcy–Bénard problem with nanofluids, has also attracted equal importance in the literature because of its importance in many fields of modern science, engineering and technology, chemical and nuclear industries and bio-mechanics. Such an instability problem was first considered by Nield and Kuznetsov [10]. Following this formalism several studies were undertaken subsequently to investigate various additional effects on the problem by the same authors and others. The details can be found in the monograph of Nield and Bejan [11]. In studying these convective instability problems, the volume fraction of nanoparticles was prescribed at the boundaries. Recently, Nield and Kuznetsov [12] pointed out that this type of boundary condition on volume fraction of nanoparticles is physically not realistic as it is difficult to control the nanoparticle volume fraction on the boundaries, and suggested an alternative boundary condition that is, the flux of volume fraction of nanoparticles is zero on the boundaries.

Studies have also revealed that nanofluids containing SiO_2 nanoparticles with ethylene glycol and water as base fluids

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Nomenclature

a	wave number
D_B	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
d	depth of the porous layer
k	thermal conductivity of the nanofluid
K	permeability of the porous medium
Le	Lewis number
l, m	wave numbers in the x- and y-directions
M	heat capacity ratio
N_A	modified diffusivity ratio
N_B	modified particle density increment
p	pressure
$\vec{q} = (u, v, w)$	nanofluid velocity
R_m	basic density Darcy–Rayleigh number
R_t	thermal Darcy–Rayleigh number
R_n	nanoparticle concentration Darcy–Rayleigh number
(x, y, z)	Cartesian coordinates
t	time
T	nanofluid temperature
T_0	temperature at the lower boundary
T_1	temperature at the upper boundary
W	amplitude of perturbed vertical component of velocity

Greek symbols

β	the coefficient of thermal expansion
ε	porosity of porous media
η	thermal expansion coefficient of viscosity
κ	thermal diffusivity of the fluid
λ_1	constant relaxation time
λ_2	constant retardation time
Λ_1	stress relaxation parameter
Λ_2	strain retardation parameter
μ	viscosity of the fluid
ω	growth rate
ϕ	nanoparticle volume fraction
ϕ_0	Reference value of nanoparticle volume fraction
Φ	amplitude of perturbed nanoparticle volume fraction
ρ	nanofluid density
Θ	amplitude of perturbed temperature

Superscripts

$*$	dimensionless variable
$\prime$	perturbed variable

Subscripts

b	basic state
f	fluid
p	particle

demonstrate a non-Newtonian behavior at low temperatures (Namburu et al. [13]). Besides, Chen et al. [14–16] and Schmidt et al. [17] also indicated the non-Newtonian rheological behavior of nanofluids. Thus, it is imperative to consider non-Newtonian effects in the study of convection in nanofluids. There exist different kinds of non-Newtonian fluids and they do not lend themselves to a unified treatment. Many of the base fluids exhibit viscoelastic behavior and hence considering viscoelastic model is more appropriate than an inelastic type of non-Newtonian model in the study of thermal convective instability in nanofluids. In general, viscoelastic instability is observed in polymer melts as well as in polymer solutions, which usually consist of a Newtonian solvent and a polymeric solute. These solutions are often highly elastic but have an essentially constant viscosity. They are known as Boger fluids and are reasonably well represented by the Oldroyd-B constitutive model (Bird et al. [18], Li and Khayat [19]). The Oldroyd-B constitutive model is adopted widely to examine the influence of elasticity on thermal convective instability. This is because the Oldroyd-B model represents adequately highly elastic (Boger) fluids, for which the viscosity remains sensibly constant over a wide range of shear rates. Besides, it is one of the simplest viscoelastic laws that account for normal stress effects which are responsible for the periodic phenomena arising in viscoelastic fluids. More importantly, almost all experimental measurements and flow visualization reported on the instability of viscoelastic flows have been conducted on Boger fluids. Comparison between theory and experiment becomes possible when the Oldroyd-B constitutive equation is used. Of course, there exist more realistic phenomenological or molecular-theory-based models (Bird et al. [18]; Tanner [20]) but they probably lead to a different stability picture (Larson [21]).

Copious literature is available on thermal convection in a layer of porous medium saturated by a viscoelastic regular fluid. Alishaev and Mirzadjanzade [22] were the first to deal with viscoelastic flows in porous media for calculations of delay phenomenon in filtration theory. Rudraiah et al. [23] studied thermal convection in a viscoelastic-fluid-saturated porous layer. A comprehensive review on non-Newtonian fluid flows and heat transfer in porous media is given by Shenoy [24]. Kim et al. [25] investigated thermal instability in a porous layer saturated with viscoelastic fluid and it is found that the overstability is a preferred mode of instability for a certain range of elastic parameters. Malashetty et al. [26] and Shivakumara et al. [27] analyzed the effects of local thermal non-equilibrium on the onset of convection in a viscoelastic-fluid-saturated porous layer. Zhang et al. [28] performed linear and nonlinear thermal stability analyses of a horizontal layer of an Oldroyd-B fluid in a porous medium heated from below. The details can be found in the book by Nield and Bejan [11].

Nonetheless, the study of thermal convective instability in a viscoelastic nanofluid saturated porous layer is comparatively of recent origin and it is still in a rudimentary stage. Sheu [29] studied the onset of convection in a horizontal layer of porous medium saturated with a viscoelastic nanofluid while Yadav et al. [30] extended this study to include the effect of rotation and variations in thermal conductivity and viscosity. In the latter paper a weakly nonlinear stability analysis has also been carried out. To make analytical progress, the volume fraction of nanoparticles is prescribed at the boundaries in the above studies. But it is believed that these conditions are difficult to visualize in practice. Under the circumstances, it is desirable to probe the implications of physically realistic boundary conditions as far as the volume fraction of nanoparticles is concerned.

The intent of the present paper is to study the onset of thermal convective instability in an Oldroyd-B type of viscoelastic nanofluid-saturated porous layer considering the flux of volume fraction of nanoparticles is zero at the boundaries as it is physically more realistic (Nield and Kuznetsov [12]). The resulting eigenvalue problem is solved numerically using the Galerkin method and the results are presented graphically.