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A probability model for fully developed annular flow in vertical pipes: Prediction of the droplet entrainment

Ri Zhang, Haixiao Liu *, Mingyang Liu

School of Civil Engineering, Tianjin University, Tianjin 300072, China

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ABSTRACT

The phase distributions and mechanical properties of annular flow can be regarded as random states. Hence the probability analysis is an appropriate method to investigate the possibilities of the relevant events and the statistic results of some characteristic parameters. In the present work, a probability model for fully developed annular flow in vertical pipes is proposed to predict the phase distributions and mechanical characteristics. The probability model works in three mechanisms. First, a vortex generation theory on energy transfer from vortexes to droplets is supposed to describe the atomization process. Second, a random walk theory is applied to track the droplet deposition on the liquid film. Third, the atomization and deposition rates are respectively related to the probabilities of droplet generation and elimination by analyzing the interaction between vortexes and droplets. Based on the knowledge of dynamic equilibrium between atomization and deposition processes, a balance equation is established to close the equation set and the representative parameters of annular flow can be solved. The new model is a statistical method and almost links all the parameters involved in annular flow. By comparing the predicted droplet entrainment with experimental data available in the literature, the present model is well verified and demonstrates advantages both in accuracy and in convenience. Furthermore, the effects on the entrainment of many parameters, including the gas flow rate, liquid flow rate, gas density, liquid density, gas viscosity, surface tension and pipe diameter, are discussed in detail.

1. Introduction

Annular flow is the most common regime in two-phase flow, which is characterized by a thin liquid film distributed along the perimeter of a conduit with a core of gas flowing in the center. This flow pattern occurs in a variety of industrial situations including phase-change heat exchangers, oil well pipelines, fossil-fired boilers and nuclear reactors. The droplet-film interaction is a complex phenomenon involving the mass, momentum and energy transfer between the film and gas core flow, and can be measured by many parameters such as the atomization rate, deposition rate, droplet size distribution, film thickness and interfacial shear-stress. The understandings of formation mechanism, morphological characteristics and physical property are essentially significant for theoretical development and engineering applications.

As a result of the importance of annular two-phase flow, numerous researchers have developed mechanistic models describing the physical processes of annular flow. The first mechanistic model for annular flow was proposed by Calvert and Williams [1], in which an analytical expression of the velocity profile in the film was obtained starting from the shear stress distribution by assuming that the mixing length distribution in the film is the same as that for a full pipe. Similarly, Willis [2] deduced the liquid film velocity profile and pressure gradient from the shear stress, while the theory was based on a force balance equation. Wallis [3] and Govier and Aziz [4] treated the annular flow in a general mechanism and many empirical relationships were adopted in their theories. The work by Hewitt and Hall-Taylor [5] on gas–liquid annular flow adopted a triangular relationship that relates three variables: film thickness, liquid film flow rate and film shear stress, which can take into account more properly the momentum and mass balances for the liquid film and the gas core. Hughmark [6] and Asali et al. [7] applied the same empirical closure relation for the film thickness to solve the two-fluid equations. Whalley [8] solved the film thickness from a formulation of the triangular relationship instead of resorting to an empirical relation.

Some more complex mechanistic models for annular flow were developed due to the promotion of computer performance. Oliemans et al. [9] illustrated the two-fluid formulation for

* Corresponding author. Tel./fax: +86 2227401510.
E-mail address: liuhx@tju.edu.cn (H. Liu).

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Nomenclature

A	area
C_D	drag force coefficient
D	pipe diameter
d_D	droplet size
e_σ	surface energy of droplet
e_v	kinetic energy of vortex
e'_v	available kinetic energy of vortex
E	entrainment
J_G J_L	superficial velocities of gas phase and liquid phase, respectively
l_d	droplet displacement
n_{d_D}	droplet density per unit length
Oh	Ohnesorge number
p	pressure
P	probability
R_a	atomization rate of droplet
R_d	deposition rate of droplet
Re_d	particle Reynolds number
t_λ	vortex lifetime
v_λ	characteristic vortex velocity
v_G v_L	mean velocities of gas phase and liquid phase, respectively
v_d^r	droplet velocity in the radial direction
v_*	friction velocity
V_v	vortex volume
V_δ	overlap volume between the liquid film and vortex
We	Weber number
y	distance away from the gas-liquid interface

Greek letters

δ	film thickness
$\Gamma(x)$	Gamma Function
λ	length scale of vortex
μ_G	viscosity of the gas phase
ρ_G ρ_L	densities of liquid phase and gas phase, respectively
σ	surface tension coefficient
τ_G	interfacial shear stress
τ_L	wall shear stress
ζ	systematic deviation coefficient

annular-mist applications in oil and gas wellbores. The model was developed from small diameter and mostly air–water data and applied to the steady state upward gas–liquid flow. Both Yao and Sylvester [10] and Alves et al. [11] also formulated mechanistic annular-mist models based on the two-fluid equations. Chen et al. [12] presented a hybrid Eulerian–Lagrangian method for modeling annular flow and improved a stochastic-probabilistic model to track the droplets in the gas core. Kishore and Jayanti [13] adopted an Eulerian–Eulerian multifluid modeling approach to research annular gas–liquid flow in a duct. An additional scalar transport equation for the mass fraction of the droplets was solved to take account of the presence of droplets. Alipchenkov et al. [14] suggested a three-fluid model of the dispersed-annular regime, which can determine the mean droplet size by introducing the equation for the number density of droplets of the dispersed phase. Rodriguez [15] performed a numerical simulation of a two-phase annular flow with the DNS (Direct Numerical Simulation) method. A VOF (Volume of Fluid) model was implemented by Liu et al. [16] to simulate the roll waves directly, which can provide self-standing information for both the gas core flow and film flow as well as the inner tube wall situations. Anglart and Caraghiar [17] reviewed the Eulerian–Eulerian and Eulerian–Lagrangian approaches for modeling annular flow and discussed their closure relationships and the pertinent conservation equations.

Almost all the mechanistic models for annular flow mentioned above need closure relationships to close the equation set. The most important closure relation in annular mist flow is the entrainment correlation, defined as the ratio of the mass flow of drops to the total mass flow of liquid. Wallis [3] defined a dimensionless ground to calculate the fraction of the liquid entrained, dependent upon the superficial gas velocity, gas viscosity, gas density, liquid density and interfacial tension. The correlation proposed by Oliemans et al. [9] is a purely empirical approach by using the Harwell data bank, which involves nine characteristic parameters and ten empirical coefficients. Ishii and Mishima [18] applied a tangent function to estimate the entrainment, which depends on the liquid Reynolds number and a modified Weber number. Nakazatomi and Sekofuchi [19] obtained an equation for entrainment by assuming that the generation of entrainment is in proportion to the dynamic pressure of the liquid phase and the inertia force of the gas phase, and is in inverse proportion to the surface tension of the liquid phase. Pan and Hanratty [20] improved the previous models [7,21] and developed an equation for entrainment resulting from a balance between the atomization rate of the liquid film and the deposition rate of droplets entrained in the gas. Different from the other entrainment correlations, the model introduced the critical liquid and gas flow rates that are needed for atomization to occur. Holowach et al. [22] developed a mechanistic model derived from the stability analysis and simplified force balances, which can directly account for the interfacial instabilities. Sawant et al. [23] improved the tangent function [18] and introduced a limiting entrainment fraction that is similar to the concept of the critical liquid flow rate [20]. Cioncolini and Thome [24] regarded the annular flow as a special form of a liquid atomization process, and applied an incomplete sigmoid function to fit the expression of entrainment based on 2460 data points.

The present work aims at developing a probability model for fully developed annular flow in vertical pipes. The new model has two distinguishing features. First, the probability model is quite appropriate to predict the statistical result of many random events, such as the phase distributions and mechanical properties of annular flow. Because the state of annular flow can be regarded as a random state, the characteristic parameters of annular flow are also stochastic variations. The proportional parameters such as the droplet entrainment and the atomization and deposition rates are predicted as the possibilities of some random events. The morphological parameters including the film thickness, mean droplet size and interfacial shear stress can be regarded as the statistic results of abundant samples. Second, the new model links almost all the parameters of annular flow rather than individually investigates each of the parameters. By relating the probabilities of droplet generation and elimination to the atomization and deposition rates, a dynamic equilibrium equation is established that can reflect the spatial distribution and the movement of droplets. Combined with the mass and momentum conservation equations, eight characteristic parameters of annular flow can be solved, including the droplet entrainment, friction velocity, film thickness, mean velocities of gas and liquid phases, pressure gradient, and wall and interfacial shear stresses. The present paper discusses the prediction of the droplet entrainment by the probability model. Furthermore, the effects on the entrainment of many parameters are also inspected in detail.