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Constructal design for a disc-shaped area based on minimum flow time of a flow system

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ABSTRACT

Based on constructal theory, a model of a "disc-point" flow system with various flow speeds is considered in this paper. The optimal constructs of the radial-pattern and first order branched-pattern discs are obtained by taking the minimum maximum flow time and minimum average flow time as optimization objectives, respectively. The results show that there exist an optimal aspect ratio of the elemental sector and an optimal elemental fraction of the high speed flow channel which lead to the minimizations of the maximum flow time and average flow time of the flow system, respectively. The optimal constructs based on these two objectives are different. When the first order fraction is 0.1 and the number of elemental tributaries is 2, the dimensionless critical radius, which determine whether the radial-pattern design or branched-pattern design for the high speed flow channels is adopted, is 3.02 for minimum maximum flow time objective and is 2.29 for minimum average flow time objective. There exists an optimal number of the elemental tributaries which leads to the double minimum dimensionless average flow time of the flow system. The constructal optimization of the flow system by taking minimum flow time as optimization objective is helpful to improve its transfer performance.

1. Introduction

Constructal theory [1–13] is a so powerful theory that it is widely used, ranging from engineering to geophysics and biology. The constructal law can be stated as [2,8]: "for a flow system to persist in time (to survive) it must evolve in such a way that it provides easier and easier access to the current that flow through it". The flow systems include various processes and devices in engineering, river networks and ocean currents in nature, tissues and organs of life bodies, etc.

Many scholars have studied various flow systems by taking different optimization objectives. Bejan [1] firstly put forward the street network model in a rectangular area where the flow of the people was located. There existed various travel speeds in the rectangular area. The optimal construct of the street network was obtained by taking the minimizations of the maximum travel time as well as the average travel time as optimization objectives, respectively. This work was the start of the constructal theory. In the heat conduction field, Bejan [14] firstly carried constructal optimization of a "volume-point" heat conduction model by taking maximum temperature difference minimization as optimization objective. The higher order assembly was assembled by the optimized last order assembly, and the heat transfer performance of the "volume-point" heat conduction system was improved. In the mass transfer field, Bejan and Errera [15] firstly studied a "volume-point" mass transfer problem with porous media by using constructal theory. The optimal construct of the rectangular element was obtained by taking maximum pressure drop minimization as optimization objective, and the mass transfer performance of the "volume-point" mass transfer system was improved. In the economics fields, Bejan et al. [16] firstly carried constructal optimization of the "area-point" economics problems based on rectangular and triangular elements by taking the minimum cost in the goods transportation. The result showed that the dendritic pattern of routes and the optimal shape of the element area could be obtained. In the electric transport field, Arion et al. [17] also extended the application of constructal theory to the design of electric power distribution networks, and the optimized tree-shaped network with a hierarchical structure of the electric power was obtained by taking the minimum discounted total cost as optimization objective. Following these work

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Nomenclature

A_p	area of the high speed flow channels, m^2
A_0	area of the elemental sector, m^2
A_1	area of the central sector, m^2
D_0	width of the high speed flow channel in elemental sector, m
D_1	width of the high speed flow channel in central sector, m
H_0	periphery half-length of the elemental sector, m
H_1	periphery half-length of the central sector, m
N	number of the peripheral sectors
n	number of the elemental tributaries
$\dot{n}''$	heat/mass/goods generation rate per unit area, W/m^2, $kg/m^2/s$, $Ton/m^2/s$
R	radius of the whole disc, m
R_0	radius of the elemental sector, m
R_1	radius of the central sector, m
t_0	maximum flow time in the radial-pattern disc, s
$\bar{t}_0$	average flow time in the radial-pattern disc, s
t_1	maximum flow time in the first order branched-pattern disc, s
$\bar{t}_1$	average flow time in the first order branched-pattern disc, s

Greek symbols

α	tip angle of the central sector, rad
ϕ_0	elemental fraction of the high speed flow channel
ϕ_1	first order fraction of the high speed flow channel
v_0	flow speed off the high speed flow channel, m/s
v_{p0}	flow speed in the high speed flow channel, m/s

Subscripts

m	minimum
opt	optimal

Superscript

$\sim$	dimensionless

mentioned above, the constructal optimizations of various flow systems were further carried out, such as transportation and material flow systems [18–21], heat transfer systems [22–35], mass transfer systems [36,37], heat and mass transfer systems [38–40], civil engineering systems and life systems.

For the heat transfer problem, Rocha et al. [32] studied the "disc-point" heat conduction model with high conductivity channels inserted in the heat generating area, and obtained the optimal constructs of the radial- and branched-pattern discs. The result showed that there existed the critical disc radius, which determined whether the radial-pattern design or branched-pattern design for the high conductivity channels was adopted.

Based on the street network model with rectangular element in Ref. [1] and the "disc-point" heat conduction model in Ref. [32], the flow system in a disc-shaped area will be discussed in this paper. Different flow speeds are assumed in the disc, and the minimum flow time is taken as the optimization objective. These are the similarities between this paper and Ref. [1]. The optimal constructs of the radial-pattern and first order branched-pattern discs will be obtained. The innovation of this paper is introducing the "disc-point" model into the constructal design of the flow system. The purpose of this paper is to search for the critical disc radius, which determines whether the radial-pattern design or branched-pattern design for the high speed flow channel is adopted. By doing this, the relationship between the flow performance and the internal complexity of the "disc-point" model can be obtained, but this relationship had not been investigated for the "area-point" model in Ref. [1].

2. Radial-pattern disc

The typical flow system studied by Bejan [1] is the transportation system with different travel speeds in the rectangular area. Actually, the flow can be heat current, mass flow, material flow as well as traffic flow, etc, and the area of the flow system can be circular, square and even erose ones. The flow system in a radial-pattern disc is shown in Fig. 1 [32]. The density of the flow (heat generation rate per unit area for heat current, mass flow rate per unit area for mass flow, goods amounts per unit area for material flow, travel people per unit area for traffic flow, etc.) is $\dot{n}''$. The flow goes through the high speed flow channels (high conductivity channel for heat current, high permeability channel for mass flow, goods channel for material flow, street for traffic flow, etc.) into the center of the disc (the heat or mass is collected, the material center or the market is located). The radius of the radial-pattern disc is R_0. There are a number (N) of the high speed flow channels with width D_0 in the R_0 disc. The flow speed is v_0 off the high speed flow channel and is v_{p0} in the high speed flow channel ($v_{p0} \gg v_0$).

According to the distribution of the high speed flow channels, the disc can be divided into a number (N) of equal sector districts, and each sector can be viewed as a fundamental element as shown in Fig. 2. When $N \gg 1$, each sector can be considered as an isosceles triangle (the base is $2H_0$, and the height is R_0), the area ($A_0 = 2H_0 \times R_0/2$) is fixed, the number of the sectors is $N = 2\pi R_0/(2H_0)$, and the aspect ratio (H_0/R_0) of the sector is free to vary. Assuming that each sector is sufficiently slender ($H_0 \ll R_0$), the flow speed direction is approximately parallel to y-direction in the v_0 area, and is approximately parallel to r-direction in the high speed flow channel.

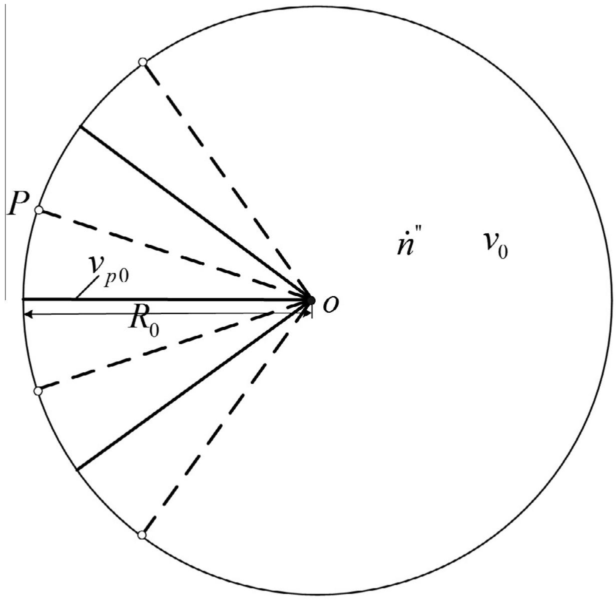

Fig. 1. Flow system in a radial-pattern disc [32].