



Anomaly of spontaneous transition to instability of liquid–vapour front in a porous medium



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ARTICLE INFO

Article history:

Received 9 December 2013

Received in revised form 31 July 2014

Accepted 1 January 2015

Keywords:

Geothermal system

Liquid–vapour front

Spontaneous transition

Rayleigh–Taylor instability

ABSTRACT

In this article, we have investigated the instability of the liquid–vapour front in a geothermal system with isothermal boundaries. A two–dimensional linear stability analysis of the isothermal basic state shows that the Rayleigh–Taylor mechanism is the dominant contributor to instability. A conditional expression for the critical modified Rayleigh number for different heat transport processes has been found. It has been shown that the spontaneous transition to instability is an artefact of neglecting thermal advection and the imposition of the phase change front to be equidistant from the liquid and vapour boundaries.

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1. Introduction

In 1883 Lord Rayleigh described for the first time the instability of a dense fluid overlying a lower density fluid in a gravitational field, which is known as the Rayleigh–Taylor instability [1]. The transitions to instability at fluid–fluid interfaces are of great interest on account of their wide range of applications. These instabilities can often occur at a liquid–vapour interface in a geothermal system [2–10]. There is much need for the better understanding of the different physical phenomena involved with liquid–vapour phase changes, and this is the focus of our study.

The term “spontaneous” transition in continuum mechanics refers to a special case of instability; when all wave numbers become unstable at the same value of the controlling parameter. The “spontaneous” transition of Il’ichev and Tsytkin [11,12] is shown to be a very unusual case, depending not only on the front position but also on the neglect of advective heat transport. Tsytkin and Il’ichev [11,12] investigated different cases of transition to instability of a stationary vertical phase change flow under the condition that conduction dominates over advection. It was shown that if the interface is equidistant from the liquid and vapour boundaries then there is a spontaneous transition to instability. A spontaneous transition to instability is a very unusual

phenomenon and so it is worth investigating more thoroughly the conditions under which it can occur. In particular, (i) is it an artefact of the relationship assumed by Tsytkin and Il’ichev [11] between the phase-change temperature and the pressure, or does it occur in other models; and (ii) is it crucial that advective heat transport is neglected?

Tsytkin and Il’ichev [7] using typical values for physical quantities, deduce that advection may be neglected if

$$K|\delta P - \rho_w g L| \ll 10^{-10} \text{N}, \quad (1)$$

where K is permeability, δP is the pressure difference across the layer, L is the characteristic length scale, g is gravity and ρ_w is the density of water. If we assume that $|\delta P - \rho_w g L|$ is of the same order of magnitude as $\rho_w g L$ (i.e., that the applied pressure difference is roughly comparable in magnitude to the hydrostatic pressure across the layer) then (1) simplifies to

$$K \rho_w g L \ll 10^{-10} \text{N}, \quad (2)$$

or, using $\rho_w = 1000 \text{ kg m}^{-3}$ and $g \approx 10 \text{ m s}^{-2}$, then (2) yields

$$KL \ll 10^{-14} \text{m}^3.$$

Thus for a sandstone with $K = 10^{-10} \text{ m}^2$, advection can be neglected only if $L \ll 10^{-4} \text{ m}$, i.e., for any large-scale aquifer advection is important. For a granitic rock with $K = 10^{-16} \text{ m}^2$, advection can be neglected only if $L \ll 10^2 \text{ m}$, so it may be negligible under these conditions.

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Nomenclature*Latin*

c_p	specific heat
g	acceleration due to gravity
H	reciprocal of Stefan number
K	permeability
k	thermal conductivity
L	thickness of the low permeable layer
l	wave number
P	pressure
q	heat flux per unit area
S	location parameter of the interface
T	temperature
t	time
x	vertical coordinate
y	horizontal coordinate

Greek symbols

ϵ	perturbation parameter
κ	thermal conductivities ratio
λ	latent heat
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density
σ	spectral parameter
σ^*	asymptotic spectral parameter
Θ	dimensionless temperature
v	fluid flow velocity
φ	porosity

Dimensionless quantities

C	specific heat ratio
E	heat capacity ratio
R	kinematic viscosity ratio
R_1	density ratio
R_2	dynamic viscosity ratio
R_3	modified Rayleigh number

Subscripts

L	liquid boundary
liq	liquid phase
m	porous medium
ref	reference quantity
S	at the phase transition front
s	porous skeleton
V	vapour boundary
vap	vapour phase
0	base state
1	perturbed state

Superscripts

$mini$	minimum
$*$	dimensionless quantity
0	base state
1	perturbed state

In the present study, the basic vertical flow is without the phase motion through the interface. We employ a simpler relation of temperature and pressure at the liquid–vapour interface ($T = T_S(P)$) by assuming a constant temperature and a continuous pressure at the front and a more complete heat transport equation. Both the pressure and the temperature profiles for the base flow are linearly distributed. The important aspect of this analysis is that we will consider a more realistic perturbed state accounting for thermal advection. The Il'ichev and Tsyppin [7,11] analysis will be studied as a special case. We will show that the transition to instability is not spontaneous as found by Il'ichev and Tsyppin [7,11], indicating that the interesting behaviour in their model is an artefact of taking a very simple model which neglected thermal advection.

2. Mathematical model

We consider a uniform, isotropic and fully fluid saturated porous layer of infinite extension bounded by two horizontal, much more permeable layers. The upper and lower highly permeable layers are filled either with vapour and liquid, respectively or liquid and vapour, respectively (see Fig. 1). In the low-permeability layer there exists a phase change front which separates the liquid phase from the vapour phase.

2.1. Governing equations

The continuity equation for incompressible flow in dimensionless form is

$$\frac{\partial u_{liq,vap}^*}{\partial x^*} + \frac{\partial v_{liq,vap}^*}{\partial y^*} = 0. \quad (3)$$

Darcy's equation is taken to hold in each phase and are presented in scaled form as

$$\left. \begin{aligned} u_{liq}^* &= -\left(\frac{\partial P_{liq}^*}{\partial x^*} + R_3\right), & u_{vap}^* &= -\frac{R_1 R_2 \kappa}{C} \left(\frac{\partial P_{vap}^*}{\partial x^*} + R_1 R_3\right), \\ v_{liq}^* &= -\frac{\partial P_{liq}^*}{\partial y^*}, & v_{vap}^* &= -\frac{R_1 R_2 \kappa}{C} \frac{\partial P_{vap}^*}{\partial y^*}. \end{aligned} \right\} \quad (4)$$

In the equations above, R_3 is the modified Rayleigh number which has been defined as

$$R_3 = \frac{K \rho_{liq}^2 c_{p,liq} g L}{\mu_{liq} k_{m,liq}},$$

where K is the permeability of the homogeneous medium and g is the acceleration due to gravity with x^* -co-ordinate increasing downwards. The modified Rayleigh number R_3 will be our key quantity for understanding the Rayleigh–Taylor instability in a geothermal system.

We will use the one-equation model to describe the heat transport in the porous medium assuming local thermal equilibrium. In dimensionless form, the equations in the liquid and vapour regions become

$$\left. \begin{aligned} E_{liq} \frac{\partial \Theta_{liq}}{\partial t^*} + u_{liq}^* \frac{\partial \Theta_{liq}}{\partial x^*} + v_{liq}^* \frac{\partial \Theta_{liq}}{\partial y^*} &= \frac{\partial^2 \Theta_{liq}}{\partial x^{*2}} + \frac{\partial^2 \Theta_{liq}}{\partial y^{*2}}, \\ \frac{E_{vap} \kappa R_1}{C} \frac{\partial \Theta_{vap}}{\partial t^*} + u_{vap}^* \frac{\partial \Theta_{vap}}{\partial x^*} + v_{vap}^* \frac{\partial \Theta_{vap}}{\partial y^*} &= \frac{\partial^2 \Theta_{vap}}{\partial x^{*2}} + \frac{\partial^2 \Theta_{vap}}{\partial y^{*2}}. \end{aligned} \right\} \quad (5)$$

It can be seen from the above equations that the energy transport is coupled with the mass transport, which introduces non-linearities. But this is not the only reason for the inherited non-linearities. The other reason is the coupling of the interface position with the heat and mass transport equation.

The most important aspect of phase change problems is the energy and mass balance at the interface of the two phases, which

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