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# Multi-mode heat transfer analysis during freezing of an encapsulated storage medium

Antonio Ramos Archibold [a,b,f], D. Yogi Goswami [a,c,*], Muhammad M. Rahman [e], Elias K. Stefanakos [a,d]

[a] Clean Energy Research Center, University of South Florida, Tampa, FL, USA
[b] Department of Mechanical Engineering, University of South Florida, Tampa, FL, USA
[c] Department of Chemical & Biomedical Engineering, University of South Florida, Tampa, FL, USA
[d] Department of Electrical Engineering, University of South Florida, Tampa, FL, USA
[e] Department of Mechanical Engineering, Wichita State University, Wichita, KS, USA
[f] Department of Mechanical Engineering, Universidad Autónoma del Caribe, Barranquilla, Colombia

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ABSTRACT

Simultaneous conduction, convection and thermal radiation have been analyzed during the freezing of a non-opaque, non-gray phase change material (PCM) encapsulated in a closed spherical container and heated at relatively high temperatures ($250 \leqslant T \leqslant 325$ °C). In contrast to the well-known conduction-dominated model of the single phase Stefan problem, in the present study the influence of the participating thermal radiation and the buoyancy induced natural convection within the melt layer is highlighted and analyzed. A two-dimensional, axisymmetric, transient model has been solved numerically. The discrete ordinate method was used to solve the equation of radiative transfer and the finite volume scheme was used to solve the equations for mass, momentum and energy conservation. The effect of additional parameters like the shell size and the external heat transfer coefficient imposed on the outer shell surface as a boundary condition have also been analyzed. It was found that the contribution of thermal radiation on the solidification process of $NaNO_3$ is to reduce the solidification time by 17% as compared with the limiting case where thermal radiation is neglected.

## 1. Introduction

Assessment of the fundamental thermal energy transport mechanisms during phase change in materials at relatively high operating temperatures has recently gained considerable attention in the solar energy research community because the urgent need to explore and develop advanced energy storage systems [1–3] based on latent heat storage in solar thermal power plants. Special attention has been devoted to the melting and freezing of thermal storage media confined in spherical or cylindrical containers to be used in packed bed and shell and tube heat exchangers respectively [4,5] The ability to quantify and appropriately predict, from a theoretical perspective, the energy contributions from conduction, convection and thermal radiation and their influence on the melting and freezing is necessary to evaluate the thermal performance and obtain accurate thermal designs of the aforementioned systems [6–9].

An extensive literature review of the reported solutions of the PCM solidification problem reveals that the most commonly analyzed model assumes that the medium is initially at the phase change temperature. Therefore, no heat transfer is allowed across the liquid phase, and the heat only flows in the solid portion. This simplified model is known as the single phase Stefan problem. Closed-form, semi-analytical solutions of the aforementioned problem has been presented by [10–17] based on the similarity variable approach, asymptotic theory and parameter-perturbation methods. Tao [18] reported a numerical solution of the single phase Stefan problem. The formulation in which the presence of temperature gradients across the liquid phase is considered has been termed the two phase Stefan problem and has been experimentally and numerically analyzed, during freezing in vertical tubes, by Sparrow and Broadbent [19] and Sparrow and Ohkubo [20] respectively. The studies conclude that initial superheating of the liquid moderately diminishes the frozen mass and the associated latent energy extraction at small times, but has a small effect on these quantities at large times. Also, good agreement was found between the numerical predictions and the experimental results.

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* Corresponding author at: Clean Energy Research Center, University of South Florida, Tampa, FL, USA. Tel.: +1 813 974 0956; fax: +1 813 974 5250.
*E-mail address:* goswami@usf.edu (D. Yogi Goswami).

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**Nomenclature**

| | |
|---|---|
| $c_p$ | specific heat at constant pressure (J/kg K) |
| $Fo$ | Fourier number $(\kappa/\rho\ c_p)(t/R_i^2)$ |
| $g$ | gravitational acceleration (m/s$^2$) |
| $Gr_R$ | Grashof number $(g\beta\Delta TR_i^3\rho^2/\eta^2)$ |
| $h_\infty$ | external heat transfer coefficient (W/m$^2$ K) |
| $h$ | sensible enthalpy (J/kg) |
| $L$ | latent heat of fusion (J/kg) |
| Nu | Nusselt number $(q_{cd}+q_{cv})R_i/\kappa(T_m-T_\infty)$ |
| $P$ | pressure (Pa) |
| $Pr$ | Prandtl number of the fluid $(\eta c_p/k)$ |
| $q_{cd}$ | conductive heat flux at the inner shell wall (W/m$^2$) |
| $q_{cv}$ | convective heat flux at the inner shell wall (W/m$^2$) |
| $q_r$ | radiative heat flux at the inner shell wall (W/m$^2$) |
| $R_i$ | inner radius of the capsule (m) |
| $Ra$ | Rayleigh number $(GrPr)$ |
| $Ste_L$ | liquid phase Stefan number $(c_{p_L}(T_o-T_m)/L)$ |
| $Ste_s$ | solid phase Stefan number $(c_{p_s}(T_m-T_\infty)/L)$ |
| $t$ | time (s) |
| $T$ | temperature (°C) |
| $T_\infty$ | free stream temperature (°C) |
| $\vartheta$ | velocity (m/s) |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | thermal diffusivity (m$^2$/s) |
| $\beta$ | thermal expansion coefficient (1/K) |
| $\delta$ | shell wall thickness (m) |
| $\eta$ | dynamic viscosity (kg/m s) |
| $\kappa$ | thermal conductivity (W/m K) |
| $\kappa_a$ | absorption coefficient (m$^{-1}$) |
| $\lambda$ | latent heat (J/kg) |
| $\rho$ | density (kg/m$^3$) |
| $\sigma$ | Stefan–Boltzmann constant, $\sigma = 5.67\times10^{-8}$ W/m$^2$ K$^4$ |

*Subscripts*

| | |
|---|---|
| $m$ | melting |
| $o$ | initial |
| $r$ | radial direction |
| $s$ | solid |
| $w$ | wall |
| $\theta$ | angular direction |
| $\Psi$ | dimensionless radiative flux at the inner shell wall $q_r/\sigma(T_m^4-T_\infty^4)$ |

An extension of the single phase Stefan problem has been presented by McCue et al. [21]. In the model, the initial temperature of the system was above the phase change temperature. However, only heat conduction was taken into account through the liquid phase. The study concludes that for small Stefan number conditions, the temperature within the inner liquid phase rapidly decreases to the phase change temperature. Consequently, heat conduction within the liquid does not affect the solidification process. Ismail and Henriquez [22] performed a parametric analysis on the single phase Stefan problem for water confined in a spherical shell. They discussed the influence of the capsule size, shell material, shell thickness and outer surface boundary condition on the PCM solidification rate. The study concludes that an increase of the Biot number at the external surface of the shell decreases the solidification time. Although the domain geometry and the boundary conditions are the same in the present analysis and the study reported in [22], the present analysis expands the previous one by including the contributions of thermal radiation and natural convection within the PCM on the net energy transport. These contributions were included in the equation of energy conservation and present a more realistic physical description of the problem.

Freezing experiments of *n*-hexadecane encapsulated in spherical shells have been reported by Chan and Tan [23]. The evolution of the solidification phase via visualization front is presented for different outer surface and initial temperatures. The study concludes that the solidification phase front progresses concentrically inwards from the colder outer surface of the sphere. Regin et al. [24] presented a numerical analysis on the solidification process of paraffin wax within a horizontal cylindrical capsule externally heated by a convection boundary condition.

The influence of the PCM volume reduction during the solidification in a partially filled spherical shell has been studied by Assis et al. [25]. In the model, the domain was initially partially filled with liquid paraffin wax, with air in the remaining volume. Numerical results and experimental observations show that an upper void space is created inside the capsule due to the PCM volume reduction during the solidification process. The study also concludes that natural convection heat transfer in the liquid phase is negligible for Stefan numbers below 1. Diffusion-controlled freezing of a micro encapsulated PCM has been analyzed by Yang and Zhao [26]. The influence of thermal radiation on the single phase Stefan problem for non-opaque media have been largely limited to planar systems [27–32] while the problem in spherical media has received little attention; especially when all the fundamental heat transfer modes are simultaneously present.

The objective of the present work is to examine the influence of simultaneous conduction, convection and thermal radiation on the freezing dynamics of a non-opaque media, with spectral dependence on its optical properties, encapsulated in a closed spherical shell at relatively high temperatures, in order to estimate the induced error in the solutions where participating thermal radiation within the PCM has been omitted. To accomplish that, a two-dimensional, axisymmetric, transient model has been solved numerically. The primary motivation of this investigation is to provide additional information about the key elements in any phase change problem, i.e., solid–liquid interface position and phase distribution at a given time after the application of a thermal driving force, particularly under operating thermal conditions related to thermal energy storage for concentrating solar power plants.

## 2. Heat transfer analysis

The considered system is schematically shown in a cross sectional view in Fig. 1. It consists of an opaque, gray and a diffuse spherical shell of inner radius $R_i$ and wall thickness $\delta$. Initially the shell inner volume is completely filled with liquid PCM at temperature $T_o$, which is higher than the storage material melting temperature $T_m$. At time $t > 0$, the outer boundary surface, whose emissivity was set to unity, is subjected to a convective boundary condition characterized by a heat transfer coefficient $h_\infty$ and a free stream temperature $T_\infty$, which is lower than $T_m$. The PCM is treated as a semitransparent, non-gray, medium being emitting and absorbing. Uniform and equal index of refraction of each phase has been assumed. The scattering of thermal radiation has been neglected in this study because of the lack of the PCM property data.

Multiple physical assumptions have been considered in the mathematical model and attention will now be turned to discuss their applicability. Experimental observations on the freezing of a low temperature PCM [23] and the natural convection heat transfer to a fluid contained in spherical containers [33] suggest that