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Chaotic and oscillatory magneto-convection in a binary viscoelastic fluid under g-jitter

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ABSTRACT

A weak onlinear analysis of double diffusive convection in an electrically conducting viscoelastic fluid layer heated from below, has been performed for the range of viscoelastic parameters where oscillatory mode exists. A uniform magnetic field has been applied vertically. Employing complex non-autonomous Ginzburg–Landau equation, the effects of various parameters on double diffusive convection is investigated. Weak nonlinear analysis reveals that the values of viscoelastic parameters have significant effect on the instability. The present study is to investigate the effect of time periodic gravity field on heat and mass transfer in the presence of external magnetic field, where controlling convection external to the system is important. It is found that the variation of Nusselt, Sherwood numbers with respect to the slow time becomes rapid upon increasing the solutal Rayleigh number Rs, Prandtl number Pr, time relaxation parameter λ and amplitude of modulation δ_2 or decreasing on Chandrasekhar number Q, diffusivity ratio Γ, time retardation parameter ε and frequency of modulation Ω. It is found that the applied magnetic field has a stabilizing effect, hence reducing heat and mass transport in the system. Further, modulated gravity field can be used either to delay or enhance the heat and mass transfer in the system. Chaotic convection under gravity modulation has also been investigated using Lorenz model.

1. Introduction

Thermal instability in a horizontal fluid layer in the presence of different physical configurations is well documented by Chandrasekhar [1] and Drazin and Reid [2]. Thermohaline convection is an important fluid dynamics property that involves motions driven by two different density gradients diffusing at different rates. In double diffusive convection, the buoyant force is affected by difference of temperature and concentrations of the fluid. This kind of example can be seen in oceanography, lakes and under ground water, chemical processes, atmospheric pollution, laboratory experiments, modeling of solar ponds (Akbarzadeh and Manins [3]), electrochemistry, magma chambers and Sparks (Huppert and Sparks [4]), Fernando and Brandt [5], formation of microstructure during the cooling of molten metals, fluid flows around shrouded heat dissipation fins, grain storage system, migration of moisture through air contained in fibrous insulations, the dispersion of contaminants through water saturated soil, crystal P growth, solidification of binary mixtures, and the underground disposal of nuclear wastes. Much work has been done on double-diffusive convection in an electrically conducted fluid layer because of its natural occurrence as above applications. Convection in planetary cores, stellar interiors and Earth's metallic core occurs in the presence of strong magnetic field. The study of double diffusive magneto convection has recently drawn the attention of oceanographers, geophysicists, astrophysicists, engineers and a host of others (Turner [6]; Rudraiah [7]). The study of magneto convection in an electrically conducting fluid layer is motivated by astrophysical and geophysical applications, relate in some way or the other to problems concerning the external constraints like rotation or magnetic field operative on two component fluid convection, in particular by observation of sunspots, Thomas and Weiss [8].

Lortz [9] was the first to consider the effect of magnetic field on double diffusive convection and clarified some of the mathematical aspects of stability criterion (Malkus and Veronis [10]) but, it was silent about the detailed study of stability analysis. Stommel et al. [11] found that, the diffusion is generally a stabilizing factor in a single component fluid. But in two component fluid, it can act to release the potential energy in the component that is heaviest at the top and make the system unstable. Pearlstein [12] analyzed

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Nomenclature

Latin Symbols

$\mathbb{A}$	amplitude of convection
$\vec{q}$	fluid velocity
a	wavenumber
δ	amplitude of gravity modulation
d	depth of the fluid layer
B	magnetic induction vector
$\vec{g}$	acceleration due to gravity
J	current density
Nu	Nusselt number
Sh	Sherwood number
P	reduced pressure
Q	Chandrasekhar number $Q = \frac{\sigma \mu_m^2 H_0^2 d^2}{\mu}$
Ra	thermal Rayleigh number, $Ra = \frac{\alpha_T g \Delta T d^3}{\nu \kappa_T}$
Rs	solutal Rayleigh number, $Rs = \frac{\beta_S g \Delta S d^3}{\nu \kappa_S}$
R_0	critical Rayleigh-number
T	temperature
ΔT	temperature difference across the fluid layer
ΔS	solutal difference across the fluid layer
t	time
(x, z)	horizontal and vertical co-ordinates

Greek Symbols

α_T	coefficient of thermal expansion
β_S	solutal expansion coefficient
χ	perturbation parameter
κ_T	effective thermal diffusivity
σ	electrical conductivity
Γ	diffusivity ratio, $\Gamma = \frac{\kappa_S}{\kappa_T}$
μ_m	magnetic permeability
Ω	frequency of modulation
ω	dimensionless oscillatory frequency
$\bar{\lambda}$	stress relaxation time
$\bar{\varepsilon}$	strain retardation time
λ	dimensionless stress relaxation time
ε	dimensionless strain retardation time
μ	dynamic viscosity of the fluid
τ_1	stress tensor
ν	kinematic viscosity, $\left(\frac{\mu}{\rho_0}\right)$
ρ	fluid density
ψ	stream function
s	slow time scale
τ	fast time scale
T'	perturbed temperature
S'	perturbed solutal concentration
$\hat{k}$	vertical unit vector

Other symbols

∇^2	$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Subscripts

b	basic state
c	critical
0	reference value

Superscripts

$\prime$	perturbed quantity
$*$	dimensionless quantity

the rotational effect on the double diffusive convection. It is found that, a non rotating layer can be destabilized by rotation, and a rotating layer can be destabilized by a bottom heavy solute gradient. Oreper and Szekely [13] found that, the presence of a magnetic field can suppress natural convective currents and that the strength of the magnetic field is one of the important factor in determining the quality of the crystal. Ostrach [14] and Viskanta et al. [15] reported a complete review on double diffusive convection. Rudraiah [7] analyzed the interaction between an externally imposed vertical magnetic field and double diffusive convection in a Boussinesq fluid.

Siddheshwar and Pranesh [16,17] analyzed the effect of a transverse magnetic field on thermal instability under temperature/gravity modulation, in a weak electrically conducting fluid with internal angular momentum. Siddheshwar and Pranesh [18] studied the role of magnetic field in natural convection driven by buoyancy and surface tension forces in a horizontal layer of an electrically conducting Boussinesq fluid layer with suspended particles. They observed that the electrically conducting fluid layer with suspended particles heated from below is more stable compared to the classical electrically conducting fluid layer without suspended particles. Ramachandran and Winter [19] investigated g-jitter and Marangoni convection on float zones and found that the effects are at pronounced at lower frequency (10^{-3} Hz) and impulsive accelerating impart to fluids with high Prandtl numbers were found to have long decay time. Kaddeche et al. [20] have investigated the buoyant convection induced between infinite horizontal walls by horizontal temperature gradient. They found that the vertical magnetic field stabilizes the instabilities more quickly than the horizontal fields, but the stabilization is only obtained up to moderate values of Hartmann number. Ryskin et al. [21] studied thermal convection in ferrofluids while treating ferrofluids as binary fluid mixtures with weak solutal diffusivity but large separation ratio. Bhadauria and Sherani [23] analyzed the onset of Darcy convection in a magnetic fluid saturated porous medium subject to temperature modulation of the boundaries. Bhadauria and Srivastava [24] studied the effect of temperature modulation on magneto-double diffusive convection, considering free-free boundaries. Bhadauria [25] studied the nonlinear double diffusive convection in an anisotropic porous medium with internal heat source. Moufekkir et al. [26] investigated numerically double diffusive convection in a square enclosure filled with a gray gas in the presence of volumetric radiation. Siddheshwar et al. [27] performed a local non-linear stability analysis of Rayleigh–Bénard magneto-convection using Ginzburg–Landau equation. They showed that the gravity modulation can be used to enhance or diminish the heat transport in stationary magneto convection.

Green [28] was the first to consider thermal instability in viscoelastic fluids, in which he restricted to the case when both bounding surfaces are free, which was carried out in terms of a two time constant model due to Oldroyd [29,30] and found that an oscillatory convective flow was possible at the onset of instability. Herbert [31] considered the stability of viscoelastic fluids in heated plane Couette flow and found that, the presence of elasticity has a destabilizing effect on the flow. Vest and Arpaci [32] reported the occurrence of overstability for the typical Bénard–Rayleigh convection of a horizontal layer of homogeneous Maxwellian fluid heated from below. Sokolov and Tanner [33] showed that overstability and exchange of stabilities is possible depending on the magnitude of the values of viscoelastic parameters. Rosenblat [7] analyzed the Rayleigh–Bénard convection of viscoelastic fluids for free boundaries, whose eigenfunctions are easily obtained