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Using Generalized Integral Transforms to solve a perturbation model for a packed bed thermal energy storage tank

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ABSTRACT

A packed bed thermal storage tank is modeled using a single phase perturbation model which leads to a single advection–convection differential equation. This equation is derived using the Generalized Integral Transforms Technique (GITT). This approach allows us to obtain an analytical solution for a major case in solar applications where the fluid mass flow rate is time dependent and the inlet temperature is constant. We obtain also a semi-analytical solution for the general case where both fluid mass flow rate and inlet temperature are time-dependent. Analytical solutions for this thermal stock permits any afterwards optimization of solar power plants.

1. Introduction

Thermal energy storage (TES) is widely used in industrial applications especially in large scale Concentrating Solar Power (CSP). In fact, the main advantage of CSP technology compared to other renewable technologies is the possibility to store the energy in thermal form. TES is a good and cheap solution to overcome the mismatch between the solar energy availability and the electricity demand. It allows a continuous energy production in spite of the intermittency of the solar source [1,2].

The most commonly used configuration for TES is the two-tank configuration [3] but some studies [3,4] pointed out that this configuration is still expensive and that the one-tank configuration with filler material so called "packed bed storage" is about 35% cheaper than the two-tank storage system, due to the reduction of the storage volume and the elimination of one tank.

The filler material used in the packed-bed storage can be a Phase Change Material (PCM) or a solid material (sensible thermocline storage). In the latter case, low cost filler materials as rocks or sand can be used. Forthermore, Bindra et al. concluded in a previous work [5] that sensible heat storage systems can provide much higher exergy recovery as compared to Phase Change Material (PCM) storage systems under similar high temperature storage conditions.

Many contributions in the literature attempted to model the dynamic temperature profile in packed-bed storage tanks. Among these works, we can find a class of "two-phase" models, that use an energy equation for each phase (solid and fluid) [5–9].

We can find also another class of equivalent "one-phase" models where the fluid and the solid are considered at a same mean temperature [10–12]. This approach does not consider the existence of a thermal non-equilibrium and consequently does not consider the heat transfer between the two phases.

To overcome this issue, Kuznetsov [13] introduced the use of the perturbation theory in 1995 and recently, in 2014, Votyakov and Bonanos [14] gave a more developed perturbation model of the storage tank.

The first class of "two-phase" models is always solved numerically in the literature. Many numeric methods are used to do so as the finite volume method [15,8] or the finite differences method [9] or the method of characteristics [7,16,17]. Numerical approaches give accurate results but they are not suitable for optimization or for long period simulations (annual or seasonal operation).

The "one-phase" model have been used by Bayon [11] and solved numerically using a finite difference scheme. The same author gave an analytical description of the solution using the CDF-logistic functions in the case of constant inlet mass flow rate and constant temperature [11]. The dependence of the CDF-logistic function parameters with the working conditions was obtained by fitting that function to numerical results.

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Nomenclature

A	volumic exchange area, m^{-1}
A^*	advection coefficient
C_p	specific heat capacity, J K^{-1} kg^{-1}
D^*	diffusion coefficient
d_p	solid particle diameter, m
h	heat transfer coefficient, J K^{-1} m^{-2}
k	thermal conduction, W m^{-1} K^{-1}
L	storage's height, m
Pe	Peclet number
Pr	Prandtl number
Re	Reynolds number
T	temperature, K
u	fluid velocity, m s^{-1}
z	spacial coordinate, m

Greek letters

α	thermal diffusivity, m^2 s^{-1}
$\delta\theta$	non-dimensional solid–fluid temperature difference
ρ	density, kg m^{-3}
τ	non-dimensional time
θ	non-dimensional temperature
ε	void fraction
ζ	non-dimensional space coordinate
ϕ	transition matrix

Subscripts

0	initial
corr	corrected
eff	effective value
f	fluid
in	inlet
s	solid
sf	solid to fluid
w	wall

Finally, some analytical solutions exist in the literature as in [13] and in [14]. Both works gave analytical solutions for perturbation models by using the Fourier method of separation of variables. However, they considered the simple case of constant inlet mass flow rate, constant inlet temperature and uniform initial temperature in the tank.

In the present study, we use the Generalized Integral Transforms Technique (GITT) [18] to solve the perturbation model of the packed-bed storage in general operation conditions: time dependent mass flow rate and temperature at the inlet of the tank and any initial temperature profile. This method allows to transform the partial differential equation to a system of ordinary differential equations by applying an integral transformation. This transformation is defined through the derivation of an auxiliary boundary-value problem. Using this approach, we give a full analytical solution for the case where the fluid mass flow rate is time dependent and the inlet temperature is constant. It permits also to derive a semi-analytical solution for the most general case: both fluid mass flow rate and inlet temperature are time-dependent.

2. Thermal model of the storage tank

2.1. Problem description

The thermocline storage system considered in this study is shown in Fig. 1. It consists of a vertical tank filled with a solid material. This layout is widely used in the literature [14,16,17]. During the heat charge process, the hot fluid enters from the top when colder fluid exits from the bottom, and conversely, during the heat discharging process, the cold fluid enters from the bottom when warmer fluid exits from the top. The tank has two fluid distributors placed at the top and the bottom that ensure an uniform flow of the fluid over the sectional area.

2.2. Two-phase model

Considering a uniform fluid velocity, a uniform and isotropic filler material, an incompressible fluid, adiabatic walls and constant material properties, the problem can be modeled by a 1D two phases model considering two volume-averaged energy equations respectively for the fluid and the solid filler [8,14,19]. These two

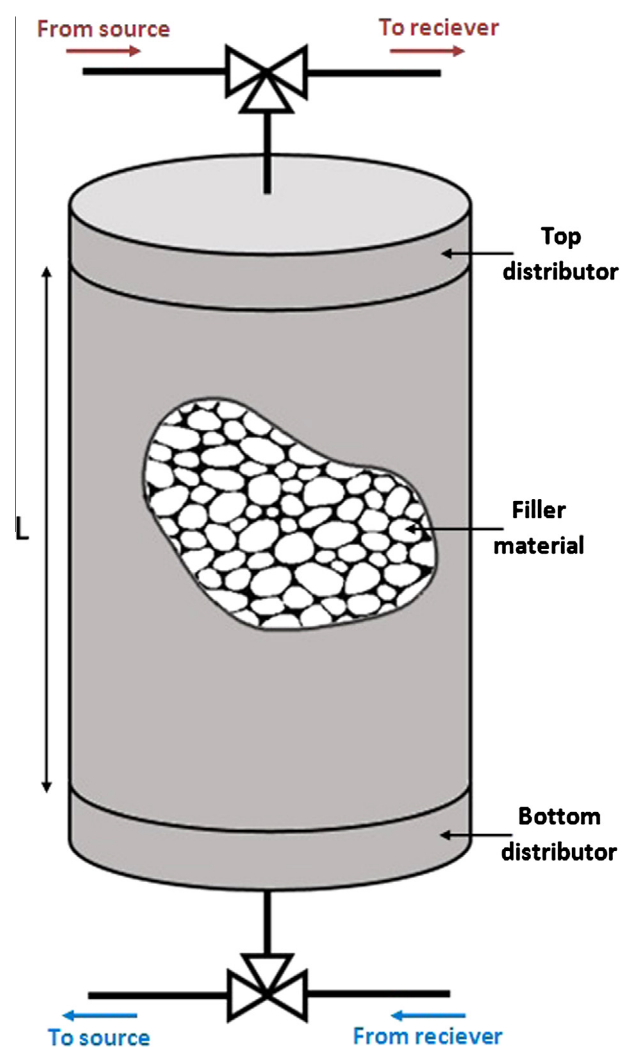

Fig. 1. Thermocline storage tank considered in this work.