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A time step amplification method in boundary face method for transient heat conduction

Fenglin Zhou [a,b], Yuan Li [a], Jianming Zhang [a,*], Cheng Huang [a], Chenjun Lu [a]

[a] State Key Laboratory of Advanced Design and Manufacturing for Vehicle Body, College of Mechanical and Vehicle Engineering, Hunan University, Changsha 410082, China
[b] Modern Jetting Department, Hunan University of Technology, Zhuzhou 412001, China

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ABSTRACT

A time domain boundary integral equation method, which is named as quasi-initial condition method, is applied in this paper to solve the transient heat conduction problem. In conventional implementations, however, this method suffers from a numerically unstable problem when the time step is small. To improve the numerical stability of the method, a time step amplification method is proposed. In the proposed method, an amplified time step is adopted to compute the temperature and the flux at the virtual time point. The boundary condition at that virtual time point is determined through a linear interpolation by the conditions at the current time step point and the quasi-initial time. Furthermore, the heat generation in the virtual time step is assumed to be constant which is the same as that in the real time step. The temperature and the flux at the current step time point are then computed through a linear interpolation over the time interval. A short but not rigorous deduction of this method is presented to show that this method is valid in solution to problems in which the temperature and the flux vary linearly respect to time. Numerical examples further demonstrate the numerical stability of the proposed method.

1. Introduction

The transient heat conduction problem widely appears in engineering problems. Many numerical methods have been proposed to solve this problem such as the finite difference method (FDM) [1,2], the finite volume method (FVM) [3,4], the finite element method (FEM) [5,6] and the boundary element method (BEM) [7–17]. Among these methods, the BEM seems to be more attractive for its dimension reduction feature. For transient heat conduction problem, BEM may be classified into two catalogs: the transformed domain method [7–9] and the time domain method [10–17]. The transformed method usually leads to an accurate result. In that method, however, it is very difficult to determine the transformation parameters which play a great important role in the numerical scheme. Moreover, for many practical problems, a large number of sampling frequencies is often required to obtain accurate solutions. Hence the numerical inverse transformation is usually very time-consuming and the accelerated techniques should be employed [7,8].

In this paper, we concern the time domain method. There are two different implementations of time domain methods. One employs the time-independent fundamental solution and the other one employs the time-domain fundamental solution. In the case of time-independent fundamental solution, the derivatives with respect to temporal variable are treated through a time domain difference scheme. In that implementation, domain integrals of the temperature are involved and the computational scale is related to the number of domain nodes. Thus, it lost the advantages of dimension reduction in the BIE method.

The numerical method employing the time-domain fundamental solution was first used by Thaler et al. in [10]. In Brebbia's work [11], this type of method is further classified into two schemes. One is named by convolution quadrature method (CQM) and the other one is the quasi-initial condition method. In the CQM, temperature and flux at each step are computed through a convolution of temperature and flux on the boundary at previous steps. If the initial temperature and the heat generation are omitted, the CQM leads to a pure boundary method. As indicated in [12–16], however, the CQM suffers from the time-consuming convolution especially in the case that a long time history is concerned. Many methods were proposed to accelerate the computation of the convolution. Gupta et al. applied an expanded fundamental solution to reduce the calculation of boundary integrals that appear in the time convolution [13]. Considering the decay of the time domain fundamental solution, Banerjee et al. developed an efficient time

* Corresponding author. Tel.: +86 731 88823061.
E-mail address: zhangjm@hnu.edu.cn (J. Zhang).

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domain convolution method in which the number of integral points was determined adaptively [14]. By combining with the boundary face method [18–22], Zhou et al. applied the efficient time domain convolution method to analyze structures with open-ended tubular holes [15]. If the initial temperature or the heat generation is not negligible, however, two methods lost their dimension reduction advances.

In this paper, we will implement the quasi-initial condition method to solve transient heat conduction problem. In this method, the temperature which is computed in the previous step is treated as the initial temperature in current step. Thus, the domain integral of this initial temperature is involved in the BIE. Compared with the CQM, however, time consuming convolution is avoided in this method. Furthermore, in our implementation, the scale of corresponding systems which should be solved is just related to the number of boundary nodes and is independent of the number of domain nodes.

As pointed out in [23–25], the quasi-initial condition method becomes numerically unstable for small time step. Iso and Onishi discussed the unstable problem in [23]. Sharp studied the unstable problem in 1D case through matrix analysis [24]. Peirce et al. studied this problem through a Fourier transformation tool [25]. In the above works, authors tried to find the pre-condition to judge if the computation was stable. In many engineering applications, small time step should be considered.

In this paper, a time step amplification method is developed to improve the numerical stability. In this method, the time step is first amplified into a larger time step. Temperatures and fluxes at the virtual time step point are then computed. Temperatures and fluxes at the actual time step point are finally computed by applying a linear interpolation scheme in the time interval. In the computation of temperatures and fluxes at the virtual time step point, the corresponding boundary condition is interpolated by the boundary condition of actual time step point and that of the initial time point. We will prove theoretically that the temperatures and fluxes obtained by the proposed method satisfy both the governing equation and the boundary condition. Since the time step which is involved in the systems is the amplified one, the proposed method can improve the numerical stability of the solution.

Three numerical examples are presented to verify the stability of the proposed method. It is worth noting that in the last example, a practical engineering problem is considered. Comparison with the available finite element software is made, showing the ability of the proposed method in practical application.

2. The time domain boundary integral equation for transient heat conduction

This section introduces the time domain boundary integral equation for the transient heat conduction. We start from the governing equation of heat conduction problem in isotropic media:

$$\begin{aligned} & k\nabla^2 u(x,t) + Q(x,t) = \rho c \frac{\partial}{\partial t} u(x,t), \quad \forall x \in \Omega \\ & u(x,t) = \bar{u}(x,t), \quad \forall x \in \Gamma_d \\ & -k\frac{\partial u(x,t)}{\partial n} \equiv q(x,t) = \bar{q}(x,t), \quad \forall x \in \Gamma_n \\ & u(x,t_0) = u_0(x), \quad \forall x \in \Omega \end{aligned} \tag{1}$$

where the domain Ω is enclosed by $\Gamma = \Gamma_d \cup \Gamma_n$ as shown in Fig. 1, material properties $k,\ \rho,\ c$ stand for conductivity, density and specific heat, respectively. $u,\ q$ denote the temperature and heat flux. Q denotes the heat generation inside the domain. $\bar{u},\ \bar{q}$ is prescribed temperature and heat flux on the boundary. n stands for the outward normal on the boundary. t_0 is the initial time. It should be noted that the material which we concern in this paper is homogeneous.

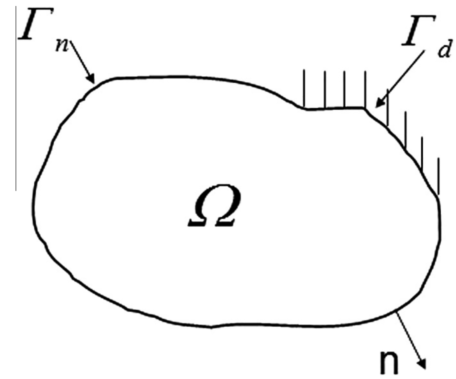

Fig. 1. Boundary conditions of heat conduction problem.

The problem can be converted into an equivalent BIE which is described as the following formulation [11]:

$$\begin{aligned} C(y)u(y,\tau) = & \frac{1}{\rho c}\int_0^\tau \int_\Gamma u(x,t) q^*(y,x;\tau,t) d\Gamma(x) dt \\ & - \frac{1}{\rho c}\int_0^\tau \int_\Gamma u^*(y,x;\tau,t) q(x,t) d\Gamma(x) dt \\ & + \int_\Omega u^*(y,x;\tau,0) u(x,0) d\Omega(x) \\ & + \frac{1}{\rho c}\int_0^\tau \int_\Omega u^*(y,x;\tau,t) Q(x,t) d\Omega(x) dt \end{aligned} \tag{2}$$

In this formula, y and x respectively stand for the field point and source point. And

$$C(y) = \begin{cases} 0 & y \in \bar{\Omega} \\ 1 & y \in \Omega \\ \frac{\theta}{2\pi} & y \in \Gamma \end{cases} \tag{3}$$

in which θ is the solid angle of the boundary at collocation point y and $\theta = \pi$ when the boundary near y is smooth. $u^*(y,x;\tau,t)$ and $q^*(y,x;\tau,t)$are the fundamental solutions which can be respectively written as

$$u^*(y,x;\tau,t) = \frac{H(\tau - t)}{(4\pi K(\tau - t))^{1.5}} e^{\frac{-r^2}{4K(\tau - t)}} \tag{4}$$

and

$$q^*(y,x;\tau,t) = -k\frac{\partial u^*(y,x;\tau,t)}{\partial n(x)} \tag{5}$$

Since the variations of temperature and heat flux with respect to both spatial variables and temporal variables are usually unknown, discretization both on space and time is required. We use a boundary face method, which is detailed in [18–22], to discretize the boundary of the considered domain. To compute the temperature and the flux step by step, a piece-wise linear Lagrange interpolation scheme together with a time-stepping scheme, which is named by quasi-initial condition scheme, are adopted.

3. Quasi-initial condition scheme for boundary integral equation

In the quasi-initial condition scheme, the temperature computed in the front step is considered as the initial condition in the current step. In each step, we first computed the temperature and flux on the boundary, and then we calculate the temperature on internal nodes. The time interval from 0 to τ can be divided into M increments of duration Δt. With the quasi-initial condition scheme, Eq. (2) has a time discretized form in the mth step: