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Characterization of space-dependent thermal conductivity for nonlinear functionally graded materials

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ABSTRACT

The aim of the present study is to characterize the unknown thermal conductivity of nonlinear functionally graded materials (FGMs) with the method of fundamental solutions (MFS) in conjunction with the Nelder–Mead simplex (NMS) method. The thermal conductivity is assumed to be a function varying exponentially with position and linearly/exponentially with temperature with the unknown coefficients. The determination of the thermal conductivity of FGMs is mathematically complicated due to its multi-parameter property, inherent nonlinearity and ill-posedness. The MFS, a well-known meshless collocation method, is used to deal with the two dimensional heat conduction problems. In the meantime, the NMS method is employed to search for an optimal solution by minimizing a functional measuring the difference between observed and the MFS-predicted temperatures under estimated parameters. Numerical examples show the feasibility, robustness and applicability of the proposed scheme.

1. Introduction

During the past decades, the investigation of functionally graded/gradient materials (FGMs) has been an active area of research. FGMs, originally proposed by Japanese researchers [1,2], are a new brand of composite materials. The volume fraction of the FGMs constituents varies gradually, providing a non-uniform microstructure with continuously graded macro-properties such as the thermal conductivity, elastic modulus and density. Thanks to their superior thermomechanical property, these novel materials have been extensively applied to a large number of structures, such as nuclear reactors [3], pressure vessels and pipes [4] and chemical plants [5].

The determination of the thermal conductivity for the FGMs encounters huge challenges due to its multi-parameter property, inherent nonlinearity and ill-posedness [6,7]. It is, therefore, essential to propose some effective numerical algorithms to solve this problem. A second-order finite difference procedure by Yueng and Lam [8] was presented for the inverse determination of the thermal conductivity in the one-dimensional heat conduction domain. Dowding, Beck and Blackwell [9] employed the finite element method to estimate the thermal parameters of a carbon–carbon composite material characterized by an orthotropic thermal conductivity. An inverse boundary element method combined with genetic algorithm [7] was developed to characterize the thermal conductivity of heterogeneous materials, where a bicubic polynomial was selected for the thermal conductivity variation. Mierzwiczak and Kołodziej [6] determined the temperature-dependent thermal conductivity in steady-state heat conduction problems for homogeneous materials. In their work, the thermal conductivity was expressed by a second order Taylor series approximation with respect to temperature, and its coefficients were determined by the method of fundamental solution (MFS).

This paper further characterizes the thermal conductivity tensor of nonlinear orthotropic FGMs with the MFS in conjunction with the Nelder–Mead simplex (NMS) method. The thermal conductivity of the nonlinear FGMs is assumed to be a function of the spatial coordinate and temperature with unspecified parameters [10], which varies exponentially with position and linearly/exponentially with temperature. The temperature field in a general orthotropic FGM is predicted via the MFS, and the optimal thermal conductivity is searched by the NMS method. The identification of the thermal conductivity is achieved merely based on the measured boundary temperature and heat flux.

The MFS, firstly proposed by Kupradze and Aleksidze in 1964 [11,12], is a simple and efficient boundary-only meshless collocation method. The solution of a problem is approximated by a linear combination of the fundamental solution of the governing equation by the MFS. The MFS is attractive to researchers because it is integration-free, easy to implement and converges rapidly [13,14]. The application of the MFS widely exists in heat

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conduction problems, including Cauchy problems [15,16], the identification of heat sources [17,18], identification of the boundary heat flux [19,20], determination of the thermal conductivity [6,21] and transient three dimensional heat conduction in FGM [22]. A comprehensive review on the application of the MFS in inverse problems can be found in Ref. [23] and references therein. In the meantime, the NMS method is employed to search for an optimal solution by minimizing a regularized functional measuring the difference between observed and the MFS-predicted temperatures under estimated parameters. The NMS method has been used extensively to estimate parameter [24,25] since its inception by Nelder and Mead [26].

The rest of this paper is organized as follows. The MFS in conjunction with the NMS method is formulated for the determination of the thermal conductivity in Section 2. Section 3 examines the efficiency, accuracy and robustness of the proposed approach through four benchmark examples. Finally some concluding remarks are summarized in Section 4.

2. Thermal characterization of the orthotropic FGMs

2.1. Nonlinear steady-state heat conduction problems

Consider a two-dimensional steady-state heat conduction problem in a nonlinear orthotropic inhomogeneous media [10] without internal heat generation:

$$(\widetilde{K}_{ij}(\boldsymbol{x},T)T(\boldsymbol{x})_{,j})_{,i} = 0 \quad \forall \boldsymbol{x} = x_1, x_2 \in \Omega \tag{1}$$

subjected to following boundary conditions:

Essential boundary conditions (prescribed temperature)

$$T(\boldsymbol{x}) = f \text{ on } \Gamma_T, \tag{2}$$

Natural boundary conditions (prescribed heat flux)

$$q(\boldsymbol{x}) = -\widetilde{K}_{ij}T_{,j}n_i = g \text{ on } \Gamma_q, \tag{3}$$

where $\widetilde{K} = \{\widetilde{K}_{ij}(\boldsymbol{x},T)\}_{1\leqslant i,j\leqslant 2}$ denotes the thermal conductivity in terms of spatial variable $\boldsymbol{x}$ and temperature T, and is a diagonal and positive-definite matrix. $T(\boldsymbol{x})$ is temperature at point $\boldsymbol{x}$ and $q(\boldsymbol{x})$ represents its heat flux. n_i is the ith-direction cosine of the unit outward normal vector to the boundary $\Gamma = \Gamma_T \cup \Gamma_q$. f and g are specified continuously differentiable functions on the related boundaries, respectively. For convenience, the spatial derivatives are indicated by a comma, such as $T_{,i} = \partial T/\partial x_i$. Moreover, repeated subscript indices stand for summation convention.

In this study, the thermal conductivity is assumed as

$$\widetilde{\boldsymbol{K}}(\boldsymbol{x},T) = \alpha(T)\boldsymbol{K}\exp(2\boldsymbol{\beta}\cdot\boldsymbol{x}),\ \alpha(T) = 1+\mu T \text{ or } \alpha(T) = e^{\mu T},\ \boldsymbol{x}\in\Omega \tag{4}$$

where μ is the parameter for the temperature-dependent term, the vector $\boldsymbol{\beta} = (\beta_1, \beta_2)$ is a graded parameter vector and the matrix $\boldsymbol{K}$ is a diagonal and positive-definite matrix with constant entries.

Before solving problem (1)–(3), we reduce the problem to a linear one through the Kirchhoff transformation [27]

$$\phi(T) = \int \alpha(T)dT. \tag{5}$$

Then Eqs. (1)–(3) are recast as

$$K_{ij}\Phi_{T,ij}(\boldsymbol{x}) + 2\beta_i K_{ij}\Phi_{T,j}(\boldsymbol{x}) = 0, \quad \boldsymbol{x}\in\Omega, \tag{6}$$

$$\Phi_T(\boldsymbol{x}) = \phi(\overline{T}), \quad \boldsymbol{x}\in\Gamma_T, \tag{7}$$

$$\varphi(\boldsymbol{x}) = -(K_{ij}\Phi_{T,j}n_i)\cdot\exp(2\boldsymbol{\beta}^T\cdot\boldsymbol{x}) = \bar{q}, \quad \boldsymbol{x}\in\Gamma_q, \tag{8}$$

where $\Phi_T(\boldsymbol{x}) = \phi(T(\boldsymbol{x}))$ and the inverse Kirchhoff transformation for temperature is

$$T(\boldsymbol{x}) = \phi^{-1}(\Phi_T(\boldsymbol{x})). \tag{9}$$

It should be noted that the inverse of $\Phi_T(\boldsymbol{x})$ in Eq. (9) can be referred to Ref. [28].

2.2. Formulation for the inverse problems with the MFS

Since it is more convenient to obtain the measurement data on the boundary, it is natural to apply a boundary-type technique to solve the aforementioned problem. The MFS, a boundary only meshless method, is employed in this study. In the MFS, the solution is approximated by a linear combination of fundamental solutions with the unknown coefficients $\{\alpha_i\}$, namely,

$$\Phi_T(\boldsymbol{x}) = \sum_{i=1}^{N}\alpha_i T_F(\boldsymbol{x},\boldsymbol{y}_i), \tag{10}$$

where $T_F(\boldsymbol{x},\boldsymbol{y}_i)$ is the fundamental solution of Eq. (6). Referred to Berger et al. [29], the fundamental solution is

$$T_F(\boldsymbol{x},\boldsymbol{y}) = -\frac{K_0(\kappa R)}{2\pi\sqrt{\Delta\boldsymbol{K}}}\exp\left(-\boldsymbol{\beta}^T\cdot(\boldsymbol{x}+\boldsymbol{y})\right), \tag{11}$$

where $\boldsymbol{x} = (x_1, x_2)$ is a field point, and $\boldsymbol{y} = (y_1, y_2)$ a source point, $\kappa = \sqrt{\boldsymbol{\beta}\cdot\boldsymbol{K}\cdot\boldsymbol{\beta}^T}$, R the geodesic distance defined as $R = R(\boldsymbol{x},\boldsymbol{y}) = \sqrt{\boldsymbol{r}\cdot\boldsymbol{K}^{-1}\cdot\boldsymbol{r}^T}$, $\boldsymbol{r} = \boldsymbol{x}-\boldsymbol{y}$. K_0 denotes the zero-order modified Bessel function of the second kind and $\Delta\boldsymbol{K} = \det(\boldsymbol{K}) = K_{11}K_{22} - K_{12}^2 > 0$. The source points $\boldsymbol{y}$ are either pre-assigned or taken to be part of the unknowns of the problem along with the coefficients $\{a_i\}_{i=1}^N$. In either case, the unknowns are determined when the approximation (10) satisfies the boundary conditions (7) and (8). For simplicity, the locations of the source points are pre-assigned and taken to be a curve similar to the real boundary hereafter. The number of source points is equal to that of collocation points.

Then the heat flux can be derived as

$$\varphi(\boldsymbol{x}) = -\sum_{i=1}^{N}\alpha_i Q_F(\boldsymbol{x},\boldsymbol{y}_i), \tag{12}$$

where

$$Q_F(\boldsymbol{x},\boldsymbol{y}) = (K_{ij}T_F(\boldsymbol{x},\boldsymbol{y})_{,j}n_i)\cdot\exp(2\boldsymbol{\beta}^T\cdot\boldsymbol{x}). \tag{13}$$

Thus the following algebraic equations can be obtained to determine the unknown coefficients $\{a_i\}_{i=1}^N$ with imposing the boundary conditions

$$\boldsymbol{A} = \begin{bmatrix} T_F(\boldsymbol{x}_{i^*},\boldsymbol{y}_j;\widetilde{K}_{ij}^k) \\ Q_F(\boldsymbol{x}_{i^\#},\boldsymbol{y}_j;\widetilde{K}_{ij}^k) \end{bmatrix}(\alpha_j) = \begin{pmatrix} \phi(\overline{T}(\boldsymbol{x}_{i^*})) \\ \bar{q}(\boldsymbol{x}_{i^\#}) \end{pmatrix} = \boldsymbol{b}, \begin{cases} i^* = 1,2,\cdots,N_1 \\ i^\# = 1,2,\cdots,N-N_1 \\ j = 1,2,\cdots,N \end{cases}. \tag{14}$$

With the coefficients vector and inverse Kirchhoff transformation Eq. (9), the temperature solution $T(\boldsymbol{x})$ of Eqs. (1)–(3) is achieved.

In the thermal characterization of the orthotropic FGMs, the parameters in the thermal conductivity $\widetilde{K}_{ij}(\boldsymbol{x},T)$ are unknown and $K_{12} = K_{21} = 0$. In addition, data from temperature measurements on Γ_q and heat flux on Γ_T are available and denoted by $Y_1(\boldsymbol{x}^*)$ and $Y_2(\boldsymbol{x}^*)$, respectively. The goal of the inverse problem is to adjust the conductivity to obtain a best fit through the observed data by minimizing the objective function:

$$J(\mathbf{K}) = J(\widetilde{K}_{ij}(\boldsymbol{x}^*,T(\boldsymbol{x}^*))) = \|T(\boldsymbol{x}^*) - Y_1(\boldsymbol{x}^*)\|_{\Gamma_q}^2 + \|q(\boldsymbol{x}^*) - Y_2(\boldsymbol{x}^*)\|_{\Gamma_T}^2, \tag{15}$$

where $T(\boldsymbol{x}^*)$ and $q(\boldsymbol{x}^*)$ are the estimated temperature and heat flux which are determined by the MFS with a given thermal conductivity $\widetilde{K}_{ij}^k(\boldsymbol{x}^*,T(\boldsymbol{x}^*))$, $\widetilde{K}_{ij}^k(\boldsymbol{x}^*,T(\boldsymbol{x}^*))$ denotes the estimated quantities at