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A fast meshless method based on proper orthogonal decomposition for the transient heat conduction problems

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ABSTRACT

In order to improve computational efficiency of meshless methods based on Galerkin weak form, a fast and efficient method based on the proper orthogonal decomposition (POD) technique for transient heat conduction problems is proposed in the paper. At the first stage of the proposed method numerical simulation results or experiment data are collected as snapshots, then singular value decomposition (SVD) is applied to obtain the optimal POD basis, subsequently POD in conjunction with meshless method is used to generate the reduced model. The efficient and accuracy of the provided algorithm are examined by three examples, and the numerical examples illustrate that the meshless methods coupled with POD technique not only keeps computational accuracy, but also brings significant computational time saving for solving transient heat conduction problems.

1. Introduction

Transient heat transfer is an important phenomenon that occurs in many engineering problems. However, the complex material properties, boundary conditions and geometrical shapes make the analysis of heat conduction problems difficult except for a few simplified cases. Thus, numerical simulation is an important ways in the study of transient heat transfer. Many numerical methods such as the finite element method (FEM), the finite difference method (FDM), and the finite volume method (FVM) have been well established over the past few decades and have been successfully applied to transient heat conduction problems [1]. However, if the object has a complex shape, much time is required in preprocessing for these methods, and in performing task such as mesh generation is time and labor-consuming. In recent years, meshless methods have emerged and been used successfully in computational mechanics and heat transfer problems. In these methods, the approximation is built only based on nodes and no predefined nodal connectivity is required, then, the removal and addition of nodes in the domain are easily performed. So far, several meshless methods have been proposed in literatures, such as smoothed particle hydrodynamics (SPH) [2], element free Galerkin (EFG) method [3,4], meshless local Petrov Galerkin (MLPG) method [5], reproducing kernel particle method (RKPM) [2], radial point interpolation method (RPIM) [6] and so on. The more details of these meshless methods can refer to [7–9].

Different meshless methods have been used for the analysis of transient heat transfer problems and a brief review is presented in the following. Singh and coworkers [10] analyzed transient nonlinear heat transfer problems in solids by EFG method. Yang and Gao [11] used radial integration BEM for transient heat conduction problems. Li and his colleagues [12] proposed the MLPG method in conjunction with the modified precise time step integration method for the analysis of transient heat conduction problems. Based on the moving Kriging interpolation, Chen and Liew [13] developed meshless local Petrov–Galerkin approach to solve transient heat conduction problems in 2-D and 3-D spaces. Shibahara and Atluri [14] applied MLPG to transient heat conduction involving in a moving heat source. Khosravifard et al. [15] presented an improved meshless RPIM for nonlinear transient heat conduction problems and implemented the method to analyze the functionally graded materials with non-homogenous and temperature-dependent heat sources. Zhang and coworkers [16] advised to use the mass lumping in EFG method for transient heat conduction problems. More applications of meshless methods for heat conduction problems can refer to [17].

Although meshless methods have a lot of advantages over FEM, they also have some disadvantages. Taking EFG method as an example, which is one of the popular meshless methods, the implementation of essential boundary condition is complicated and the computational time is more than that of FEM. Especially it has high computational cost for transient problems as compared to FEM,

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Nomenclature

$\boldsymbol{P}(\boldsymbol{x})$	complete polynomial basis	n	number of neighbor points
$\boldsymbol{a}(\boldsymbol{x})$	vector of unknowns	α	the dimensionless size of the influence domain
$\boldsymbol{A}(\boldsymbol{x})$	matrix of computation	ρ	the distance between two adjacent nodes
$\boldsymbol{B}(\boldsymbol{x})$	matrix of computation	δ_{ij}	Kronecker delta function
$\boldsymbol{N}(\boldsymbol{x})$	shape functions	T	temperature
$w_i(\boldsymbol{x})$	weight function	G	heat source
$u^h(\boldsymbol{x})$	the moving least squares approximation function	k_x	thermal conductivity for x direction
k_y	thermal conductivity for y direction	q	normal heat flux
$\boldsymbol{n}$	outward surface normal	c_p	material specific heat
T^0	initial temperature	Γ_b	Dirichlet boundary
T_b	prescribed temperature on Dirichlet boundary	Γ_q	Neumann boundary
Δt	time step	$\boldsymbol{M}$	heat capacitance matrix
$\boldsymbol{K}$	heat conductance matrix	$\boldsymbol{F}$	load vector
m	dimension of the system equations	d	number of the snapshots
$\boldsymbol{T}_{snap}$	matrix of snapshots	$\boldsymbol{U}$	orthogonal matrix
V	orthogonal matrix	$\boldsymbol{\varphi}$	orthogonal eigenvector
λ	eigenvalue	$\boldsymbol{\Phi}$	optimal POD basis matrix
I	unit square matrix	$\hat{\boldsymbol{M}}$	reduced system matrix
$\hat{\boldsymbol{K}}$	reduced system matrix	$\hat{\boldsymbol{F}}$	reduced load vector
k	number of optimal POD basis		

which limits its application seriously. Consequently, any contribution to reduce the computational cost of meshless method can be regarded as an important progress [15].The main reasons that cause low computational efficiency of EFG are shown as follows: (i) There have a lot of matrices inverse and multiplication operation for computing shape function. (ii) It requires higher order Gaussian integral to ensure the computational accuracy. (iii) In every background cell for each Gauss point, it needs to be decided for which nodes the Gauss point contribute, that is, node search procedure is involved in. (iv) It needs special and complex method to impose essential boundary conditions. (v) The bandwidth of system-equations matrix which obtained by EFG is usually larger than that of FEM.

To avoid these deficiencies, in the paper we will present a very simple technique to simplify the imposition of essential boundary conditions, and introduce the proper orthogonal decomposition (POD) technique to generate the reduced model. POD is a powerful technique for low-order approximation of some high dimensional processes, which is also known as principal component analysis (PCA), Karhunen–Loeve Decomposition (KLD) or singular value Decomposition (SVD). Several contributions on equivalence and connection among these three methods can refer to [18,19].

Using the POD technique, a small sample of system response vectors known as snapshots are generated, commonly from experimental data or calculating data of high-dimensional systems, then some information is extracted from the snapshot set in the form of basis, and the approximation is obtained through the projection of a full scale discretized model onto the subspace spanned by the basis that yields low dimensional reduced models [20]. In this way, computational cost can be greatly reduced.

The literature on the application of POD is vast, and the paper makes no attempt to give a complete review of the relevant references. A detailed overview of POD can refer to [18,21]. Though POD widely used in the computation of statics, fluid dynamics, structural dynamics, etc., it is mainly applied to perform the principal component analysis and search the main behavior of a dynamic system [22]. In the past few decades, the POD technique has been used in the numerical solution to construct some reduced models. So far the POD technique has been used in the finite difference method [22], the finite element method [23,24], and the finite volume method [25] and so on. However, to the best of our knowledge, there are no published results when POD is used to reduce the classical EFG method for transient heat conduction problems. Therefore, in the paper we apply the POD technique to study the EFG method for solving transient heat conduction problems and establish a reduced EFG formulation with lower dimensions and high enough accuracy for transient heat conduction problems.

The article is organized as follows. In Section 2, the fundamental principle of the EFG method is briefly reviewed, in which we extend the nodal influence domain of the EFG method to arbitrary convex polygon. In Section 3, the implementation of the EFG method for heat conduction problems in heterogeneous media is expressed. In Section 4, a brief introduction to POD is provided, meanwhile, we use POD in conjunction with EFG to construct a reduced model. In Section 5, numerical examples are presented to demonstrate the computational accuracy and efficiency of our method. In Section 6 the article ends with concluding remarks.

2. Element free Galerkin method

2.1. Moving least square approximation

In the solving domain Ω, according to moving least square (MLS) theory, the approximate function of $u(\boldsymbol{x})$ can be expressed as follows[4,6]:

$$\boldsymbol{u}^h(\boldsymbol{x}) = \boldsymbol{P}^T(\boldsymbol{x})\boldsymbol{A}^{-1}(\boldsymbol{x})\boldsymbol{B}(\boldsymbol{x})\boldsymbol{u} = \boldsymbol{N}^T(\boldsymbol{x})\boldsymbol{u} \tag{1}$$

where $\boldsymbol{P}(\boldsymbol{x})$ is a vector of basis functions that consist of complete polynomial, $\boldsymbol{N}(\boldsymbol{x})$ is a MLS shape function, $\boldsymbol{u}$ is an unknown vector. $\boldsymbol{A}(\boldsymbol{x})$ and $\boldsymbol{B}(\boldsymbol{x})$ are matrices defined as follows:

$$\boldsymbol{A}(\boldsymbol{x}) = \sum_{i=1}^{n} w_i(\boldsymbol{x})\boldsymbol{P}(\boldsymbol{x}_i)\boldsymbol{P}^T(\boldsymbol{x}_j) \tag{2}$$

$$\boldsymbol{B}(\boldsymbol{x}) = [w_1(\boldsymbol{x})\boldsymbol{P}(\boldsymbol{x}_1), w_2(\boldsymbol{x})\boldsymbol{P}(\boldsymbol{x}_2), \ldots, w_n(\boldsymbol{x})\boldsymbol{P}(\boldsymbol{x}_n)] \tag{3}$$

In Eqs. (2) and (3), n is the number of weight function $w_i(\boldsymbol{x}) = w(\boldsymbol{x} - \boldsymbol{x}_i) > 0$.