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Conjugate heat transfer of backward-facing step flow: A benchmark problem revisited

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ABSTRACT

In this paper a benchmark solution for the conjugate heat transfer of backward-facing step flow is computed using the stream function vorticity formulation. The numerical solution is obtained using the multidomain Boundary Element Method. A significant difference was found when comparing the results with the prior benchmark solution computed by Kanna and Das (2006). Similar disagreement has also been reported in the work done by Teruel and Fogliato (2013). The new benchmark temperature and Nusselt number values were obtained using Richardson extrapolation to zero-sized mesh. The presented results have excellent agreement when compared to the third numerical code.

1. Introduction

The backward-facing step (BFS) problem is one of the more important benchmark problems used within computational fluid dynamics (CFD). BFS research started with the well-known experimental and numerical work of Armaly et al. [3]. This paper serves as a CFD validation case for many articles, for example Gartling [4] and Erturk [5] more recently. It is natural to expect that BFS geometry is also used as a heat transfer benchmark. Soon after Gartlings work two non-isothermal and non-conjugate benchmarks were published by Dyne and Heinrich [6] and Choudhury [7]. The first BFS conjugate heat transfer benchmark case was set up by Kanna and Das [1], as reported in their work.

The mentioned benchmark of Kanna and Das [1] was used within the validation process of our Boundary Element Method (BEM) for CFD. Unfortunately significant disagreement was found during this comparison. The disagreement was confirmed by using ANSYS CFX as a third CFD code. After detailed literature searching, another article from conference proceedings written by Teurel and Fogliato [2] was discovered with practically the same results as ours. In the work published recently by Seddiq et al. [8], this benchmark case is used for validation of the lattice Boltzmann method for model heat transfer at the solid–fluid interface. They found good agreement when comparing the Nusselt number at the fluid–solid interface. Unfortunately only one comparison was shown. However, these two articles confirmed that this benchmark case is still under investigation and that the confusion is still present. The motivation of this paper was to clarify the benchmark results.

There are many new research fields confirming that the BFS geometry and heat transfer is still actual and that this benchmark could be used within the code validation process. In the work of Kherbeet et al. [9], for example, BFS geometry is used in the nanofluid studies. Within the field of MHD flow there are novel works by Pekmen and Sezgin [10] and Yazdani and Yagoobi [11]. There are also numerical simulations of both fields; MHD and nanofluid, as published in the work of Sheikholeslami et al. [12]. These are only a few recent works dated in 2014. There are also many variations of BFS geometry used regarding heat transfer problems. BFS geometry with baffles was used in the work of [13]. Next, the natural convection within the BFS geometry variation was dealt with in [14]. The effect of the rotating cylinder on the forced heat convection of ferrofluid over BFS is shown in [15]. These mentioned articles are only a few among many dated in the year 2014.

The papers structure is as follows. After the Introduction, the Problem definition is stated in Section 2. In Section 3 the numerical method is presented briefly. Next, the numerical code validation is performed using an analytical solution for the conjugate heat transfer in Section 4. The aim of Section 5 is to present the accuracy of new benchmark numerical solution using a mesh independency study. The results are compared to other authors in Section 6. The paper finishes with the conclusions.

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Nomenclature

b	thickness of the solid slab [m]
$dx,\ dy$	mesh size of uniform mesh along (x, y) axes [m]
h	step height [m]
H	channel height [m]
$k_{f,s}$	thermal conductivity of the fluid and solid [W/m K]
k	thermal conductivity ratio, k_s/k_f
n_j	unit normal direction to the boundary element
Nu	local Nusselt number
$\overline{Nu}$	average Nusselt number
Pr	Prandtl number
Re	Reynolds number for the fluid
t	non-dimensional time
T	temperature
u	arbitrary field function used in general transport equation
$u^\star$	fundamental solution used in BEM integral equation
$v_x,\ v_y$	non-dimensional velocity components along (x, y) axes
$x,\ y$	non-dimensional Cartesian co-ordinates

Greek symbols

$\alpha_{f,s}$	diffusivity of the fluid and solid [m^2/s]
Γ	boundary of solution domain used in BEM integral equation
θ	dimensionless temperature
ψ	dimensionless stream function
ω	dimensionless vorticity
Ω	solution domain used in BEM integral equation

Subscripts

$*$	dimensional quantity
f	fluid
s	solid
w	wall, interface between fluid and solid

2. Problem definition

The geometry and the boundary conditions for the conjugate BFS problem are identical to the referenced work of Kanna and Das [1], and shown in Fig. 1. The hydrodynamic boundary conditions considered there and here are identical with those of Gartling [4]. The step height h is exactly defined as half of the channel height H. The Reynolds number is defined using the channel height $H = 1$ and average inlet velocity $\bar{u} = 1$.

The heat transfer boundary conditions are defined by the fluid inlet temperature 0 and solid bottom wall constant temperature 1. All other walls are adiabatic where the zero temperature derivative in normal direction is prescribed, see Fig. 1. In the outlet the adiabatic condition is set as well. In the reference work of Kanna and Das [1] many cases were computed using 4 parameter study: Reynolds number (Re), Prandtl (Pr) number, solid slab height (b) and solid fluid conductivity ratio (k). In order to reduce the number of cases, only k is varied in the presented article, while other problem parameters are fixed to the next values: $Re = 800,\ Pr = 0.71$ and $b = 4h$.

The governing equations for an incompressible laminar flow are written using the stream function vorticity formulation, the same as in [1,4]. The equations are written in transient non-dimensional forms exactly the same as in the mentioned referenced work of [1] using the same notation.

Stream function equation ψ

$$\nabla^2\psi = -\omega. \tag{1}$$

Vorticity equation ω

$$\frac{\partial\omega}{\partial t} + \frac{\partial(v_x\omega)}{\partial x} + \frac{\partial(v_y\omega)}{\partial y} = \frac{1}{Re}\nabla^2\omega, \tag{2}$$

where v_x is the velocity in x direction computed as $v_x = \partial\psi/\partial y$ and v_y as $v_y = -\partial\psi/\partial x$.

Energy equation within the fluid region

$$\frac{\partial\theta_f}{\partial t} + \frac{\partial(v_x\theta_f)}{\partial x} + \frac{\partial(v_y\theta_f)}{\partial y} = \frac{1}{RePr}\nabla^2\theta_f, \tag{3}$$

where θ_f is the non-dimensional temperature and Pr the Prandtl number.

Energy equation within the solid region

$$\frac{\partial\theta_s}{\partial t} = \left(\frac{\alpha_s}{\alpha_f}\right)\frac{1}{RePr}\nabla^2\theta_s, \tag{4}$$

where α_s and α_f are diffusivities for the solid and fluid regions respectively. The explanation for the non-dimensional form of Eq. (4) follows. The second term has non-dimensional diffusivity containing Re and Pr. The dimensional form of the solid energy equation is $\frac{\partial T^\star}{\partial t^\star} = \alpha_s\nabla^2 T^\star$. Since the time step is common for all equations, time is non-dimensionalised by $t = \frac{t^\star\overline{v_x}}{H}$ which leads to $\frac{\partial\theta}{\partial t} = \alpha_s H\overline{v_x}\nabla^2\theta_s$ and finally the non-dimensional diffusivity within the solid region as $\frac{\alpha_s}{H\overline{v_x}} = \frac{\alpha_s}{\alpha_f}\frac{1}{RePr}$. In this Benchmark problem only the steady state solution is considered. The left hand side of Eq. (4) is zero and this equation is reduced to the Laplace equation

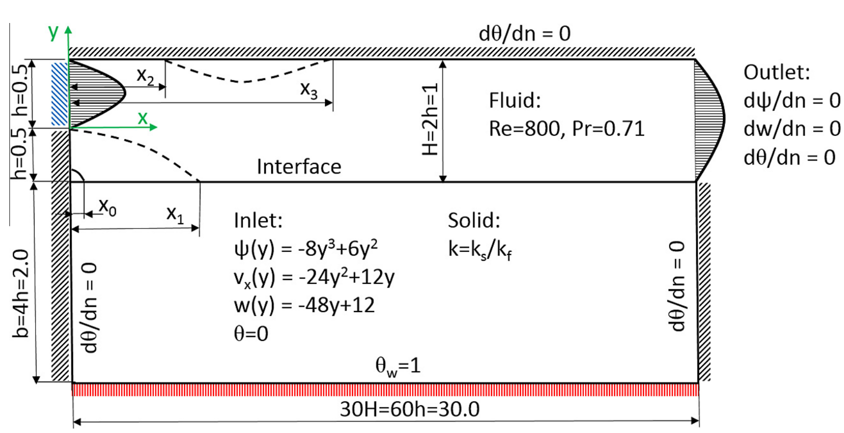

Fig. 1. Geometry and boundary conditions for conjugate backward-facing step problem.