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Constructal entransy dissipation rate minimization for triangular heat trees at micro and nanoscales

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ABSTRACT

The "volume-point" heat conduction model with triangular heat trees is considered in this paper. Several high conductivity channels are distributed in the heat generating body, and the size effects are present in these channels. The minimization of the entransy dissipation rate (EDR) is chosen as the optimization objective. The optimal constructs of the triangular heat trees at different scales are obtained by using the local optimization and global optimization constructal design methods, respectively. The results show that the optimal construct at micro and nanoscales is evidently different from that at convectional scale. The heat conduction performance of the triangular heat trees based on local optimization constructal design method can be further improved compared with that based on the global one. The optimal constructs of the triangular second order assembly (TSOA) with minimum EDR and minimum maximum temperature difference (MTD) are evidently different, and the same conclusion can be obtained for the comparison between the optimization objectives of EDR and entropy generation rate (EGR).

1. Introduction

Constructal theory [1–6] has been put forward for almost twenty years, and various design problems have been solved by using this theory. One of the typical elements of the constructal problems is the rectangular element, and various investigations are implemented based on this element model [7–12]. Except for the rectangular element, another typical element is the triangular element. Neagu and Bejan [13] firstly studied the electronic cooling problem with leaf-shaped element by using constructal theory, and reduced the global thermal resistance of the construct by 33% compared with the rectangular element. They also firstly considered a more practical triangular element with triangular high conductive pathway, and pointed out that this practical triangular element was only 6% higher than that of the leaf-shaped element. Ghodoossi and Egrican [14] studied the triangular shaped electronics with rectangular high conductive pathway by using exact method, and obtained the optimal shapes of the triangular heat generating body and rectangular high conductive pathway. Wei et al. [15] further carried out constructal optimization of this model with minimum entransy dissipation rate (EDR), and reduced the average temperature difference (ATD) of the triangular shaped electronics. Zhou et al. [16] optimized the structure of a triangular solid–gas reactor with minimum entropy generation rate (EGR), and reduced the entropy generation of the reactor. Ghodoossi and Egrican [17,18] used the cost minimization and revenue maximization models to optimize the "area to point" flow problems. The results showed that this model could only predict the shape of the flow areas, and the higher global maximum revenue could be obtained by using rectangle-in-triangle structure. Zhou et al. [19] further used the triangular element model to optimize the economic transport problem, and the result showed that the transport cost could be reduced if the angle constraint was relaxed.

To search for the essence of heat transfer, Guo et al. [20–22] introduced a new physical quantity, and named it as "entransy". With the entransy concept, they proposed an entransy dissipation extremum principle and defined an equivalent thermal resistance (ETR). Han and Guo [23] and Zhu et al. [24] further studied the physical mechanism and the experimental analogy of the entransy. Henceforth, various heat transfer problems [10,15,25–44] were studied based on the extremum principle of entransy dissipation.

Could the thermal conductivity of a material still be constant when its length scale reduced to a certain degree? This was a heat transfer problem considered size effect at micro and nanoscales [45–47]. Plentiful of works have been done in this field [48–61]. Specially, considering the size effect on the heat conduction,

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Nomenclature

A	area of the triangular body, m^2
c_v	constant volume specific heat, J/kg/K
D	width of the high conductivity channel, m
$\dot{E}_{h\phi}$	entransy dissipation rate per unit volume, W K/m^3
E_{vh}	entransy, J K
$\dot{E}_{vh\phi}$	entransy dissipation rate of a volume, W K
H_0	base of the triangular element, m
H_1	width of the first order assembly, m
H_2	width of the second order assembly, m
k_b	thermal conductivity at convectional scale, W/K/m
k_x	thermal conductivity of the high conductivity material, W/K/m
k_0	thermal conductivity of the low conductivity material, W/K/m
$\tilde{k}$	dimensionless thermal conductivity
L_0	height of the triangular element, m
L_1	length of the first order assembly, m
L_2	length of the second order assembly, m
M	mass, kg
n_1	number of the triangular elements
n_2	number of the first order assemblies
Q_{vh}	thermal capacity, J
q	heat generation rate, W
q'''	volumetric heat generation rate, W/m^3
$\tilde{R}_h$	dimensionless equivalent thermal resistance
$\dot{S}_{gen0}$	entropy generation rate of the triangular element, W/K
T	temperature, K
$T_{\min}$	temperature of the heat sink, K
x	horizontal coordinate, m
y	longitudinal coordinate, m

Greek symbols

ΔT	temperature difference, K
λ	length bound, m
ϕ	volume fraction of the high conductivity material
∇T	temperature gradient, K/m

Subscripts

m	minimum
mm	double minimum
opt	optimal
0, 1, 2	element, first order assembly, second order assembly

Superscripts

b	elemental conduction regime for $D_0 > \lambda$
bb	first order conduction regime for $D_0 > \lambda$ and $D_1 > \lambda$
bbb	second order conduction regime for $D_0 > \lambda, D_1 > \lambda$ and $D_2 > \lambda$
n	elemental conduction regime for $D_0 \leqslant \lambda$
nb	first order conduction regime for $D_0 \leqslant \lambda$ and $D_1 > \lambda$
nbb	second order conduction regime for $D_0 \leqslant \lambda, D_1 > \lambda$ and $D_2 > \lambda$
nn	first order conduction regime for $D_0 \leqslant \lambda$ and $D_1 \leqslant \lambda$
nnb	second order conduction regime for $D_0 \leqslant \lambda, D_1 \leqslant \lambda$ and $D_2 > \lambda$
nnn	second order conduction regime for $D_0 \leqslant \lambda, D_1 \leqslant \lambda$ and $D_2 \leqslant \lambda$
$\sim$	dimensionless

Gosselin and Bejan [53] considered the "volume-point" heat trees with rectangular element at micro and nanoscales, and the result showed that the optimal constructs of the rectangular heat trees were greatly influenced by the size effect exhibited in the high conductivity channel (HCC). Daneshi et al. [60] investigated the optimal structures of the radial, branched, loop and hybrid micro and nano-scale conductive pathways in a disc-shaped body, and compared the performances of these pathways with different methods. Chen et al. [61] considered a "disc-point" heat conduction model at micro and nanoscales, and obtained the critical disc radius to design the pattern of the HCCs different from that at convectional scale.

On the basis of the models with triangular element at convectional scale [14] and rectangular element at micro and nanoscales [53], a triangular element model at micro and nanoscales will be considered in this paper. The construct of the triangular heat trees will be optimized with minimum EDR. Performance comparisons of the triangular heat trees with different optimization objectives and different constructal design methods will be performed.

2. Definition of entransy dissipation rate [20]

Entransy reflects the heat transfer ability of an object [20], which is defined as

$$E_{vh} = \frac{1}{2}Q_{vh}U_h = \frac{1}{2}Q_{vh}T \tag{1}$$

where Q_{vh}(=Mc_vT) is the thermal capacity (M, c_v and T are the mass, constant volume specific heat and temperature of an object, respectively), and U_h is the thermal potential. The entransy dissipation rate per unit volume is given as [20]

$$\dot{E}_{h\phi} = -q'' \cdot \nabla T \tag{2}$$

where q'' is the vector of the heat flux, and ∇T is the gradient of the temperature. For the heat conduction in steady state, $\dot{E}_{h\phi}$ can be written in another form, i.e.,

$$\dot{E}_{h\phi} = E_{h,in} - E_{h,out} \tag{3}$$

where $E_{h,in}$ and $E_{h,out}$ are the entransy input and entransy output of the object, respectively.

The EDR of the whole heat conduction body is

$$\dot{E}_{vh\phi} = \int_v \dot{E}_{h\phi} dv = \int_v |q'' \cdot \nabla T| dv \tag{4}$$

where v is the volume. The ETR can be given as [20]

$$R_h = \dot{E}_{vh\phi}/\dot{Q}_h^2 \tag{5}$$

where $\dot{Q}_h$ is the total heat flow rate. The ETR defined based the concept of entransy is not only suitable for one-dimensional heat conduction problem, but also suitable for multi-dimensional one. When $\dot{Q}_h$ is fixed, one can see that the minimizations of EDR and ETR are equal to each other, and the construcal optimization of triangular heat trees in this paper belongs to this kind of problem.

3. Optimization of triangular element

As shown in Fig. 1, a typical triangular element ($H_0 \times L_0 \times 1/2$) uniformly generates heat in the low conductivity material (thermal conductivity k_0) [14]. The heat generation rate of the element is q. The base H_0 and the height L_0 of the triangular element and the width D_0 of the HCC all have the same meanings as discussed in the Ref. [14].

Different from Ref. [14], the size effect is taken into account when the value of D_0 reaches to micro and nanoscales. The symbol k_x is used to represent the thermal conductivity of D_0 channel