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Characteristics of thermal convection in a rectangular channel with an inner cold circular cylinder

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ABSTRACT

Based on three-dimensional numerical simulations, results are presented for natural convection in a rectangular channel with an inner cold circular cylinder. The Prandtl number is 0.7 and the Rayleigh number is changed in the range of 1×10^3 to 1×10^6. The rectangular channel is heated from the bottom wall of the channel and cooled from the top wall. The adiabatic thermal boundary condition is implemented at the vertical side walls of the rectangular channel. A low-temperature isothermal boundary condition is applied at the surface of the cylinder. The radius of the inner circular cylinder is changed in the range of 0.1–0.4*L*, where *L* is the height of the rectangular channel. By changing the radius of the cylinder, we investigate the effect of the inner cold circular cylinder on thermal convection and heat transfer in the space between the cylinder and the rectangular channel. With respect to the radius and Rayleigh number of the cylinder, the thermal and flow field is categorized into six regimes: steady symmetric two-dimensional convection, steady asymmetric two-dimensional convection, steady symmetric three-dimensional convection, steady asymmetric three-dimensional convection, time periodic convection, and aperiodic convection. The map of thermal and flow regimes is presented as a function of the radius and Rayleigh number of the cylinder. This paper presents detailed analysis results for the isotherms, vortical structure, boundary layer thicknesses, and Nusselt numbers and includes a comparison of the results for a rectangular channel without an inner cylinder.

1. Introduction

The convective flows concerned with Rayleigh–Benard convection are studied in many scientific and engineering fields. Scientific applications include astrophysical and geophysical phenomena such as sunspot behavior, mantle behavior, and atmospheric circulation. Thus, Rayleigh–Benard convection attracts interest in a wide range of sciences, including geology, oceanography, climatology, and astrophysics [1]. Rayleigh–Benard convection is also a fundamental mechanism for momentum, heat, and mass transfer in engineering systems [2]. Some examples include fluid flow in solar towers, furnace designs for uniform temperature, natural cooling in nuclear reactors, cooling of electronic devices, cooling in submarines, and eco-friendly building design using passive cooling. As mentioned above, Rayleigh–Benard convection is widely applicable to engineering equipment and scientific research. Thus, fundamental and theoretical research is necessary to understand and use Rayleigh–Benard convection. However, for complex geometry, theoretical and experimental approaches face certain difficulties in observing thermal and fluid flow in the system. Thus, Rayleigh–Benard convection in complex geometry has been investigated by numerous researchers using various numerical schemes to analyze and control the thermal and fluid flow in the system.

One of the most important objectives in the design of engineering devices is to enhance the efficiency of heat transfer. During efficient heat transfer, mutual energy transport between the body surfaces and working fluid is critical. Thus, several researchers have investigated the effect of an inner bluff body on the heat transfer and plume dynamics in an enclosure. According to previous studies conducted by Moukalled and Acharya [3], Ha et al. [4], Lee et al. [5,6], Lee and Ha [7,8], and Angeli et al. [9], the characteristics of thermal and flow fields in the enclosure were changed according to the thermal condition and the size of the bluff body in the enclosure.

Most previous studies on thermal convection in an enclosure with an inner body were investigated by using two-dimensional analysis, owing to the limitations of computational performance and the difficulty of numerical modeling [10]. Few studies have examined thermal and fluid flow in a three-dimensional enclosure with an inner body. However, as reported by Chan and Banerjee

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Nomenclature

Symbols

AC	aperiodic convection
f_i	momentum forcing
g	gravitational acceleration [m/s^2]
H	height of rectangular channel [m]
h	heat source or sink
W	width of rectangular channel [m]
L	longitudinal length of rectangular channel [m]
Nu	Nusselt number
P	dimensionless pressure
Pr	Prandtl number
q	mass source or sink
R	radius of circular cylinder [m]
Ra	Rayleigh number
S^2	strain rate of velocity gradient tensor
SA2C	steady asymmetric two-dimensional convection
SA3C	steady asymmetric three-dimensional convection
SS2C	steady symmetric two-dimensional convection
SS3C	steady symmetric three-dimensional convection
T	temperature [K]
TC	time-periodic convection
t	dimensionless time
U_{ref}	convective velocity
u_i	dimensionless velocity vector
x_i	Cartesian coordinates system

Greek letters

α	thermal diffusivity [m^2/s]
β	thermal expansion coefficient [K^{-1}]
δ_{i2}	Kronecker delta
δ_t	thermal boundary layer thickness
δ_v	viscous boundary layer thickness
λ_2	lambda-2 criteria for vortical structure
η	Kolmogorov scale
ν	kinematic viscosity [m^2/s]
θ	dimensionless temperature
ρ	density [kg/m^3]
τ_p	period of the fluctuation of thermal and flow fields
Ω^2	rotation rate of velocity gradient tensor
ξ	grid spacing

Subscripts

b	bottom wall
c	cold
cyl	cylinder
h	hot
i, j	tensor notation
ref	reference quantity
t	top wall

Superscripts

$*$	dimensional value

Overbar

-	surface-averaged quantity

[11], when the Rayleigh number is high, it is difficult to accurately analyze thermal and fluid flow in an enclosure because the convection rolls are dependent on all three spatial coordinates. Although many studies have examined thermal convection in an enclosure with no inner body that is heated from the bottom surface and cooled from the top surface, little information is available on the effect of the existence of an inner circular cylinder on the thermal convection in a rectangular channel heated from below and cooled from above. Thus, the objective of the present study is to investigate the effect of a cold inner circular cylinder on thermal convection in a rectangular channel with a vertical temperature gradient. To determine the factors affecting thermal convection in a rectangular channel with an inner circular cylinder, three-dimensional numerical simulations are conducted for a wide range of Rayleigh numbers and radii of the cylinder.

2. Computational details

2.1. Numerical methods

In this study, to investigate the effect of an inner circular cylinder on thermal convection in a rectangular channel, the immersed boundary method [12,13] was used to assign the viscous and thermal boundary conditions on the surface of the circular cylinder. The immersed boundary method has been used by some researchers [14–17] to investigate thermal and fluid flow in the space between an inner body and an enclosure. When the size of the circular cylinder is changed, the immersed boundary method is simpler and more efficient to implement than classical approaches such as body-fitted curvilinear grids.

For unsteady three-dimensional incompressible viscous flow and thermal fields, the governing equations in which the immersed boundary method is applied are the continuity, momentum, and energy equations in their nondimensional forms, which can be expressed as follows:

$$\frac{\partial u_i}{\partial x_i} - q = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \sqrt{\frac{Pr}{Ra}}\frac{\partial^2 u_i}{\partial x_j \partial x_j} + \theta\delta_{i2} + f_i, \tag{2}$$

$$\frac{\partial \theta}{\partial t} + u_j\frac{\partial \theta}{\partial x_j} = \frac{1}{\sqrt{RaPr}}\frac{\partial^2 \theta}{\partial x_j \partial x_j} + h. \tag{3}$$

In this study, except for the density in the buoyancy term, all fluid properties are assumed as constant. The Oberbeck–Boussinesq approximation is used for the density in the buoyancy term. The dimensionless variables in these equations are defined as

$$t = \sqrt{\frac{g\beta(T_h - T_c)}{H}}t^*, \quad x_i = \frac{x_i^*}{H}, \quad u_i = \frac{u_i^*}{U_{ref}} = \frac{u_i^*}{\sqrt{g\beta H(T_h - T_c)}},$$
$$P = \frac{P^*}{\rho U_{ref}^2} = \frac{P^*}{\rho g\beta H(T_h - T_c)}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \tag{4}$$

where g, β, H, and ρ represent the gravitational acceleration, volume expansion coefficient, height of the rectangular channel, and density, respectively; U_{ref} represents the convective velocity, defined as $\sqrt{g\beta H(T_h - T_c)}$. The superscript $*$ in Eq. (4) represents the dimensional variables; t is the dimensionless time; T is the dimensional temperature; x_i is the dimensionless Cartesian coordinate; u_i is the corresponding dimensionless velocity component; P is the dimensionless pressure; and θ is the dimensionless temperature. The dimensionless temperature for the hot wall was 0.5 and that for the cold wall was -0.5. The above nondimensionalized formula results in two dimensionless parameters: $Ra = g\beta H^3(T_h - T_c)/\nu\alpha$