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A thermo-mechanical model for flows in superposed porous and fluid layers with interphasial heat and mass exchange

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ABSTRACT

In this paper, we develop a thermo-mechanical model for flows in superposed porous and fluid layers with interphasial heat and mass exchange. This model is based on a mixture-theoretic formalism, according to which, the fluid and the solid phases are treated as two coexisting but open thermodynamic continua that interact with each other. As such, each phase is endowed with its own set of thermodynamic variables and conservation laws. In particular, each phase is assigned with its own temperature field, thereby allowing for thermal non-equilibrium between the two phases. Constitutive equations for all dissipative and relaxation phenomena occurring in both phases are derived by exploiting the constraints imposed by the entropy axiom when applied to the entire mixture. This model is valid for both compressible and incompressible flows. Herein we also derive its low-Mach number approximation, which is substantially simpler and, therefore, more convenient for flows where compressibility effects are negligible. The efficacy of the proposed model and the effect of thermal non-equilibrium between the two phases are examined via direct numerical simulations of natural convection in a horizontal channel consisting of a porous layer and a superposed pure-fluid domain.

1. Introduction

In the present paper we are concerned with the modeling and simulation of flows in superposed porous and fluid layers with heat transfer and interphasial mass exchange. Such flows are encountered in numerous technological and geophysical applications and have attracted particular attention over the years. The major challenges in the study of such flows is the presence of macroscopic interfaces between the porous and pure-fluid regions and the coexistence of fluid and solid phases in the porous regions.

Most often, the presence of interfaces is treated via the two-domain approach, according to which separate governing equations are prescribed on the porous medium and on the pure-fluid domain. This approach requires the prescription of matching conditions that express continuity of mass and energy fluxes, and balance of forces at the interface. Typically, governing equations for the porous regions are derived via volume averaging methods. This method essentially amounts to modifying the equations of fluid motion to account for the presence of the solid matrix and then averaging these equations over space. The literature on the modeling of porous media flows following the two-domain approach and on volume-averaging methods is quite extensive; see, for example, [1–11] and references therein. A wealth of information and numerous additional references can also be found in the textbooks [12,13].

An alternative option, less frequently used, is the single-domain approach; see, for example [13–20]. This amounts to deriving, via volume-averaging or a mixture-theoretic formalism, a single set of governing equations that is simultaneously valid in both the porous and the pure-fluid domains, thus eliminating the need for matching conditions at the interface.

Typically, in the two-domain approach, the interface is represented as a sharp discontinuity so that the flow structures in its vicinity are not resolved. A notable exception is the transition region model of [21], in which the interface is modeled as a transition layer instead. By contrast, according to the single-domain approach, the flow structures in the vicinity of the interface are resolved; the same also holds for the model of [21]. Despite the conceptual simplicity that the two-domain approach offers, the implementation of interface conditions for unsteady flows is quite challenging, especially from the computational point of view. For this reason, in our view, the single domain approach is better adapted for numerical simulations of unsteady flows.

In numerous previous studies of thermal convection in domains with superposed fluid and porous layers, it has been assumed that

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the fluid and the solid matrix are in thermal equilibrium. Therefore, a single equation, describing the energy balance of the mixture, is needed to determine the temperature inside the porous region (one-equation model). Within this context, various authors have studied the stability of thermal convection in superposed fluid and porous layers; see, for example, [22–33] and references therein. More recently, the authors of [34] presented Reynolds-averaged simulations for statistically stationary turbulent heat transfer in such domains.

However, there are applications, such as devices for rapid heat transfer etc, where the thermal-equilibrium hypothesis might no longer be valid. In this case, two energy equations are needed: one for the fluid phase and a separate one for the solid matrix. This increases significantly the complexity of the corresponding mathematical models. Nonetheless, due to their growing importance in technological applications, the study of convection in absence of thermal equilibrium has attracted considerable interest in recent years. Thus far, the bulk of the published literature has been devoted to domains that are completely covered by a porous medium; for a literature survey, see [13,14]. By comparison, there are few available publications that examine flows in domains with superposed fluid and porous layers. An influential contribution to this field is [35] in which the authors provided a set of governing equations based on volume-averaging and the two-domain approach and thermal boundary conditions at the interface; see also [4]. Subsequently, steady-state convection problems in domains partially covered by porous media have been analyzed in [36–39]. Nonetheless, numerical simulations of unsteady, developing flows in absence of thermal equilibrium have yet to appear in the literature.

In this paper we develop a thermo-mechanical model for the flows of interest, based on the one-domain approach. This choice is motivated by the fact that, as mentioned above, this approach is better adapted for the numerical study of unsteady flows. To this extent, we adopt a mixture-theoretic formalism, according to which both the fluid and the porous solid are treated as two separate and identifiable continua that occupy the same space and are in thermodynamic non-equilibrium with each other. As such, each continuum constituent is endowed with its own set of thermodynamic variables and is assigned its own set of balance laws. Also, the porosity (fluid volume fraction) is introduced as a field variable that measures the density of volume occupied by the fluid. Then, constitutive relations for the interaction between the two phases and for all dissipative phenomena occurring in each phase are derived via exploitation of the constraints imposed by the entropy-inequality axiom. It should be noted that the modeling of porous-media flows based on mixture-theoretic formalisms has a long history. Over the years, various models have been developed by employing different theories of non-equilibrium thermodynamics; see, for example, [16–20,15], and references therein.

Our modeling, however, differentiates from earlier works in several aspects. First, we follow a particular mixture-theoretic formalism, namely the one of [40], which is a generalization of the classical theory of irreversible processes to immiscible mixtures whose constituents are in thermal non-equilibrium. Second, the proposed model treats both thermal non-equilibrium between phases and heterogeneous reactions, while being valid for both compressible and incompressible fluid flows. Third, the limiting case of incompressible flows is derived via a formal asymptotic expansion, commonly known as "low-Mach number approximation", instead of assuming *a priori* that the fluid's thermodynamic pressure or density are constant.

The low-Mach number approximation is valid for flows where compressibility effects are negligible and is applicable to many practical applications and natural phenomena. An important simplification of this approximation is that the fluid momentum and energy equations are decoupled. Also, the constitutive expressions for interphasial mass exchange simplify considerably.

In this paper, we discuss in more detail the properties of the low-Mach number approximation for the special but important case of non-reacting flows (without interphasial mass exchange), and we also elaborate on the issue of thermal condition for the porous matrix at the interface. Finally, we present results from numerical simulations of unsteady natural convection in a channel. The objectives of this numerical study is to test the efficacy of the proposed model and its numerical discretization for thermal convection and to gain insight on the effect of thermal non-equilibrium between the fluid and the porous solid. To the best of our knowledge, such simulations have not appeared in the literature yet.

The article is organized as follows. In Section 1 we present the derivation of the thermo-mechanical model and in Section 2 we derive its low-Mach number approximation. Then, in Section 3 we discuss the properties of the model for the case of non-reacting flows. In Section 4 we describe the numerical method and other numerical aspects of our simulations. Finally in Section 5, we present and analyze the results of our numerical study of transient thermal convection.

2. Derivation of the mathematical model

Let $\Omega \subset \mathbb{R}^3$ be an open and bounded domain that contains both porous and pure-fluid regions. The porosity distribution $\phi(\boldsymbol{x}, t)$ is introduced as a concentration parameter that measures the density of volume occupied by the fluid. According to its axiomatic definition, [41,42], $\phi(\boldsymbol{x}, t)$ is a probability density function defined in Ω via the Radon–Nikodym theorem and takes values in the interval $(0, 1]$.

Next, let $\Omega_p \subset \Omega$ be the union of the open sub-domains that are covered by the porous material, $\Omega_p = \{\boldsymbol{x} \in \Omega : \phi(\boldsymbol{x}) < 1\}$. In the same manner, let $\Omega_f \subset \Omega$ be the union of the sub-domains covered by the pure fluid, $\Omega_f = \{\boldsymbol{x} \in \Omega : \phi(\boldsymbol{x}) = 1\}$. Therefore, $\Omega = \Omega_f \cup \Omega_p$. Also, let S_Ω denote the boundary between Ω_p and Ω_f. Since Ω_p is an open subset of Ω, then along S_Ω we have that $\phi(\boldsymbol{x} \in S_\Omega, t) = 1$. In the case of a sharp interface between porous and pure-fluid regions, S_Ω coincides with the interface. In the case of a smooth interface (with finite thickness), S_Ω represents the end of the interface at the side of the pure fluid.

The assumptions upon which our model is based are the following.

(i) Each phase is modeled as a continuum thermodynamic system.
(ii) The two thermodynamic continua are immiscible but occupy the same space. In particular, they fill completely the space that they occupy (saturation condition).
(iii) The two thermodynamic continua are open to each other and at non-equilibrium.
(iv) The mass, momentum and energy exchanges between the two continua are pure, *i.e.* their sum must vanish.
(v) The skeleton (or matrix) of the porous material is assumed to be a rigid solid of zero velocity and constant mass density. Also, the fluid is assumed to be simple and isotropic.
(vi) The postulate of phase separation holds. In other words, irreversible phenomena associated with only one phase do not depend on the variables of the other phase.

Since the two thermodynamic systems are open and at out of equilibrium, they can interact with each other. These interactions are in the form of mass, momentum, and energy exchanges, and are denoted by $\mathcal{M}$, $\boldsymbol{f}$, and $\mathcal{E}$, respectively. In particular, mass exchange can occur due to heterogeneous reactions or phase change, while momentum exchange occurs via the action of interphasial forces.