International Journal of Heat and Mass Transfer 88 (2015) 181–202

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

MHD mixed convection in a lid-driven cavity filled by a nanofluid with sinusoidal temperature distribution on the both vertical walls using Buongiorno's nanofluid model

Hillal M. Elshehabey [a,b,*], Sameh E. Ahmed [b]

[a] Mathematics Department and CEMAT, Instituto Superior Técnico, Universidade de Lisboa, Lisboa 1049-001, Portugal
[b] Mathematics Department, Faculty of Science, South Valley University, Qena 83523, Egypt

ARTICLE INFO

Article history:
Received 22 September 2014
Received in revised form 14 April 2015
Accepted 14 April 2015
Available online 14 May 2015

Keywords:
Inclined magnetic field
Nanofluid
Lid-driven cavity
Finite volume method
Thermophoresis
Brownian motion
Non-uniform heating

ABSTRACT

In the present contribution, a numerical investigation is presented to study Buongiorno's nanofluid model for MHD mixed convection of a lid-driven cavity filled with nanofluid. A sinusoidal temperature and nanoparticle volume fraction distributions on both vertical sides is considered where the horizontal walls are kept adiabatic. The cavity is permeated by an inclined uniform magnetic field and the effects of Brownian motion and thermophoresis are incorporated into the nanofluids model. An accurate collocated finite volume method is employed to discretize the governing partial differential equations after converting them to a non-dimensional form using a suitable transformation variables. Comparisons with previously published work are performed and excellent agreement is obtained. The computation is carried out for wide ranges of the Hartmann number Ha $(0 \leqslant Ha \leqslant 100)$, buoyancy ratio Nr $(0.1 \leqslant Nr \leqslant 1)$, thermophoresis number Nt $(0.1 \leqslant Nt \leqslant 1)$, Brownian motion parameter Nb $(0.1 \leqslant Nb \leqslant 1)$, Lewis number Le $(1 \leqslant Le \leqslant 10)$, Prandtl number Pr $(0.054 \leqslant Pr \leqslant 10)$, inclined magnetic field angle γ $(0^o \leqslant \gamma \leqslant 3\pi/2)$, Amplitude ε $(0 \leqslant \varepsilon \leqslant 1.5)$, phase angle ξ $(0^o \leqslant \xi \leqslant 3\pi/4)$ and Richardson number Ri $(0.001 \leqslant Ri \leqslant 100)$. The obtained results are presented in terms of the streamlines, isotherms and nanoparticles volume fraction contours as well as local Nusselt number. Results demonstrate that, the presence of an inclined magnetic filed in the flow region leads to lose the fluid movement. Also, the fluid flow is dominated by the movement of the upper wall in the case of the highest values of the buoyancy ratio.

1. Introduction

Mixed convection heat transfer and fluid flow in cavities with moving lid are important subjects of investigation due to their effect on many engineering applications and nature phenomena especially with nanofluids and MHD. The most usage of the mixed convection flow can be found in atmospheric flows, solar energy storage, heat exchangers, lubrication technologies, drying technologies and cooling of the electronic devices. On the literature survey of this subject; MHD mixed convection with joule heating effect in a lid-driven cavity with a heated semi-circular source was considered by Rahman et al. [1] where the finite element method was used in the solution of the normalized governing equations. It was found that the average Nusselt number can be decreased with the increasing of the Rayleigh number in the presence of Joule effect. The magnetic field can be a good control parameter for heat transfer and fluid flow. Grosan et al. [2] performed a numerical investigation for steady magnetohydrodynamic free convection in a rectangular enclosure filled with a fluid-saturated porous medium and with internal heat generation. Their results indicated that, the horizontal magnetic field is most effective in suppressing the convection flow for a stronger magnetic field in comparison with the vertical magnetic field. Öztop et al. [3] studied the MHD mixed convection in a lid-driven cavity with corner heater. From which they could recorded that, the temperature distribution inside the cavity mostly stems from the right side of corner due to impinging air. Also, thermal boundary layer becomes higher with increasing of Hartmann number and isotherms fit with the corner. Hydro-magnetic combined convection in a lid-driven cavity with sinusoidal boundary conditions on both sidewalls was investigated by Sivasankaran et al. [4]. Their study showed; the heat transfer rate increases on increasing the

* Corresponding author at: Mathematics Department and CEMAT, Instituto Superior Técnico, Universidade de Lisboa Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal.

E-mail address: hilal.hilal@sci.svu.edu.eg (H.M. Elshehabey).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2015.04.039
0017-9310/

Nomenclature

Symbol	Description
$\mathbf{B}$	uniform magnetic field
$\mathbf{B}_0$	magnitude of the magnetic field $\mathbf{B}$
$\mathbf{B}_x, \mathbf{B}_y$	magnetic field component in x, y-axes
D_B	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
e_x, e_y	unit vectors along the coordinate axis
$\mathbf{F}$	electromagnetic force
g	gravitational acceleration, m/s^2
Gr	Grashof number
H	length of of the cavity, m
Ha	Hartmann number
$\mathbf{J}$	electric current density
k	thermal conductivity, W/m K
K_r	thermal conductivity ratio
Le	Lewis number
Nu	local Nusselt number
Nu_a	average Nusselt number
Nb	Brownian motion parameter
Nr	buoyancy ratio
Nt	thermophoresis number
P	dimensionless pressure
p	pressure, Pa
Pr	Prandtl number
Ra	Rayleigh number
Re	Reynold number
Ri	Richardson number
T	temperature, K
U	non-dimensional velocity component in X-direction
U_p	constant reference speed of the top wall
u	dimensional velocity component in x-direction, m/s
V	non-dimensional velocity component in Y-direction
v	dimensional velocity component in y-direction, m/s
X	non-dimensional coordinate in horizontal direction
x	Cartesian coordinate in horizontal direction, m
Y	non-dimensional coordinate in vertical direction
y	Cartesian coordinate in vertical direction, m

Greek Symbols

α	thermal diffusivity, m^2/s
β	coefficient of thermal expansion, K^{-1}
γ	magnetic field inclination angle, degree
σ	electric conductivity, W/m K
ε	amplitude
ξ	phase angle ξ
θ	dimensionless temperature
ϑ	enclosure inclination angle, degree
$\boldsymbol{\varphi}$	electric potential
φ	solid volume fraction of nanoparticles
ϕ	dimensionless solid volume fraction of nanoparticles
ψ	stream function, m^2/s
Ψ	dimensionless stream function
Ω	vorticity function
μ	dynamic viscosity, kg/m s
ρ	density, kg/m^3
$\triangledown$	Laplace operator (dimensional) $\triangledown = \partial^2/\partial x^2 + \partial^2/\partial y^2$
$\blacktriangledown$	Laplace operator (dimensionless) $\blacktriangledown = \partial^2/\partial X^2 + \partial^2/\partial Y^2$

Subscripts

c	cold wall
E	east face of the control volume
f	pure fluid
f_0	reference fluid
L	left side of the cavity
N	north face of the control volume
P	cell location of the control volume
R	right side of the cavity
S	south face of the control volume
W	west face of the control volume
w	wall

amplitude ratio. Also, the flow behavior and heat transfer rate inside the cavity are strongly affected by the presence of the magnetic field. Kefayati et al. [5] investigated the Lattice Boltzmann simulation of MHD mixed convection in a lid-driven square cavity with linearly heated wall. The results demonstrated that the augmentation of Richardson number causes heat transfer to increase, as the heat transfer decreases by the increment of Hartmann number for various Richardson numbers and the directions of the magnetic field. Effects of moving lid direction of the same problem was considered by Al-Salem et al. [6]. It was recorded that direction of lid is more effective on heat transfer and fluid flow in the case of mixed convection than it is the case in forced convection. Moreover, heat transfer is also decreased with increasing of magnetic field for all studied parameters. Nasrin [7] investigated the influences of physical parameters on mixed convection in a horizontal lid-driven cavity with an undulating base surface using finite element method. The results concluded that the wavy lid-driven cavity can be considered an effective heat transfer mechanism at larger wavy surface amplitude, as well as the number of waves and cavity. Chatterjee [8] studied MHD mixed convection in a lid-driven cavity including a heated source whereas Chatterjee et al. [9] investigated magneto convective transport in a vertical lid-driven cavity including a heat conducting square cylinder with joule heating. Chatterjee's study recorded that the heat transfer rate and bulk fluid temperature are both found to heave increasing function of mixed convective strength. Moreover, it was observed in the second study that the presence of the heat conducting obstacle causes more heat transfer to take place and, furthermore, the square shaped solid object improves the heat transfer rate in comparison to a similar size circular object. Numerical study on mixed convection in an inclined lid-driven cavity with discrete heating can be found by Sivasankaran et al. [10]. In which the high heat transfer was found at cavity inclination angle of $\gamma = 30^\circ$ in the buoyancy convection dominated regime when the heater is located at the middle of the cavity. Khanafer and Aithal [11] considered the problem of laminar mixed convection flow and heat transfer characteristics in a lid driven cavity with a circular cylinder. Their results showed that, for dominant mixed convection, the average Nusselt number increases with an increase in the radius of the cylinder for various Richardson numbers. Ahmed et al. [12] investigated the effect of non-uniform heating on both sidewalls on MHD mixed convection in an inclined lid-driven cavity with opposing thermal buoyancy force. It was indicated that the rate of heat transfer along the heated walls is enhanced on increasing either Hartmann number or inclination angle. Furthermore, the non-uniform heating on both walls provides higher heat transfer rate than non-uniform heating of one wall. A study for mesoscopic simulation of double-diffusive mixed convection of pseudoplastic fluids in an enclosure with sinusoidal boundary condition was done by Kefayati [13] where Selimefendigil and Öztop [14] considered the problem of numerical investigation and dynamical analysis of mixed convection in a