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Use of a fictitious Marangoni number for natural convection simulation

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ABSTRACT

In this paper, a method based on the use of a fictitious Marangoni number is proposed for the simulation of natural thermocapillary convection as an alternative to the traditional effective diffusivity approach. The fundamental difference between these two methods is that the new method adopts convective mass flows in simulating natural convection. Heat transfer in the natural convection simulation is calculated through the mass transport. Therefore, empirical Nusselt numbers correlations required in the effective diffusivity method are eliminated. This represents a clear advantage in the context of design studies where flexibility in varying the geometry unconstrained by the availability of appropriate correlations is highly desirable. The new method is demonstrated using a simple geometrical model. An analytical expression of the fictitious Marangoni number associated with convection between vertical plates is derived and a computational fluid dynamics (CFD) simulation is performed to study the efficacy of the proposed method. The results show that the new method can approximate real natural convection quite accurately and can be used to simulate the convective flow in complex, obstructed or finned structures where the specific Nusselt correlation is not known.

1. Introduction

Natural convection in enclosed cavities is of great importance in many engineering and scientific applications such as energy transfer, boilers, nuclear reactor systems, energy storage devices, etc. In the design of such systems numerical simulation using computational fluid dynamics (CFD) and experimental testing of prototypes are extensively used. However, these methods are not well suited to activities such as parametric analysis due to their time-consuming nature and high cost [1].

Natural convection analysis often involves complex simulations. Such simulations entail a set of relaxation factors to converge and no easy way to find the relaxation factors except through continuous trials demanding significant computational resources. This frequently dissuades thermal analysts and designers from attempting 3D simulations. In order to overcome this problem, the traditional approach is to use an effective diffusivity term (effective thermal conductivity) to convert effects of convection into pure conduction [2,3]. The fluid within an enclosure behaves like a fluid the thermal conductivity κ of which is modified by an effective thermal conductivity κ_{eff} as $\kappa_{\text{eff}} = \kappa \cdot \mathbf{Nu}$, with the Nusselt number **Nu** being determined by an appropriate correlation. This provides a challenge to engineers when they are designing a complex or novel system. The engineers must have knowledge of the appropriate Nusselt number correlation relationship for the specific geometry, such as finned structures; however, these correlations are often not available. This then motivates us to find an alternative approach that does not require knowledge of Nusselt number correlations.

In this paper, an alternative approach is proposed for natural convection simulations in which the *momentum equation* is modified and then the mass flow represented by using a fictitious Marangoni term in the stress tensor inducing thermocapillary currents. The heat transfer is then the result of this mass flow. In the next section the theoretical background behind the proposed approach will be presented. Although prior knowledge of Marangoni convection is not essential to understand the material presented in the next section, the interested reader is referred to the text by Kuhlmann and Rath [4] for further information about fundamental Marangoni theory and to recent research outputs [5–14] and the book by Lappa [15] to obtain an overview of thermal convection and the state of the art.

2. Theoretical background

2.1. The fictitious Marangoni approach (FMA)

Let us start by considering the Navier–Stokes equation, which has, in presence of a gravitational field, the following tensorial form

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Nomenclature

A_c	fin base-area
C_1	constant
c_p	specific heat capacity
D	diffusion coefficient
g_i	gravitational acceleration
h	channel or plate length
h_f	heat transfer coefficient
l	plate width or fin length
m	fin temperature profile parameter
Ma	Marangoni number
n	exponent
Nu	Nusselt number
P	fin perimeter
Ra	Rayleigh number
s	plate spacing
T	temperature
v	velocity
w	mass flow per unit width
z	length coordinate

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
γ^*	fictitious surface tension
κ	thermal conductivity
ρ	density
σ	surface tension
η	dynamic viscosity
η_{fin}	fin efficiency

Subscripts

a	ambient value
c	cold
eff	effective value
f	fluid
h	hot
i, j, k	coordinate directions
s	surface

$$\left(\frac{\partial v_i}{\partial t} + v_k \frac{\partial v_i}{\partial x_k}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_k}\{\sigma'_{ik}\} + g_i \tag{1}$$

where v is the velocity, p is the static pressure, σ'_{ik} is the viscous stress tensor (described below), and g_i is the gravitational body force per unit volume.

The viscous stress tensor is given by

$$\sigma'_{ik} = \eta\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} - \frac{2}{3}\delta_{ik}\frac{\partial v_l}{\partial x_l}\right) \tag{2}$$

where η is the dynamic viscosity and δ_{ik} is the Kronecker delta.

Now let us derive the equation describing the natural convection. For the sake of simplicity, we will assume the fluid is incompressible. This assumption implies that the variation of density due to variation in pressure may be neglected. We can express the variations in temperature, density and pressure as functions of small variations dT, $d\rho$ and dp, respectively. This is the well-known Boussinesq approximation (for buoyancy). Introducing this into the Navier-Stokes equation (Eq. (1)), results in the following expression [16]:

$$\left(\frac{\partial v_i}{\partial t} + v_k \frac{\partial v_i}{\partial x_k}\right) = -\frac{1}{\rho}\frac{\partial dp}{\partial x_i} + \frac{\partial}{\partial x_k}\{\sigma'_{ik}\} - \beta dT g_i \tag{3}$$

where, with our assumption that the fluid is incompressible, i.e. $div \cdot u = 0$, the stress tensor is simplified as

$$\sigma'_{ik} = \eta\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right) \tag{4}$$

Let us now consider the situation of a boundary condition that must be satisfied at the boundary between the fluid and the walls, when surface-tension forces are taken into account. If we assume that the surface-tension coefficient γ is not constant over the surface (in our case because of temperature variation), then a force tangential to the surface is developed $\frac{\partial \gamma}{\partial x_i}$, and the stress tensor then becomes

$$\sigma'_{ik} = \eta\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right) + \frac{\partial \gamma}{\partial x_i} \tag{5}$$

Now, our objective in this paper is to define a fictitious surface-tension gradient, $\partial\gamma/\partial x_i$, which emulates the buoyancy potential, $\beta dT g_i$. Although there are several ways in which to do this, perhaps the following analogy is easiest in applying this approach to cases involving plates and finned structures.

In laminar, fully developed, two-dimensional (2D) flow between parallel plates (see Fig. 1), the pressure drop is given by [16]

$$\left.\frac{dp}{dx_i}\right|_{\mathrm{loss}} = -2\frac{\sigma'_{ik}}{s} \tag{6}$$

where s is the distance between the plates and the stress tensor is given by

$$\sigma'_{ik} = \eta\left(\frac{\partial v_i}{\partial x_k}\right)_{x_k=0} \tag{7}$$

or, considering the velocity profile between the parallel plates [17],

$$\left.\frac{dp}{dx_i}\right|_{\mathrm{loss}} = -12\frac{\eta w}{\rho s^3} \tag{8}$$

where w is the mass flow rate per unit of width.

For natural convection flow, this flow resistance is balanced by the buoyant potential [17] given by

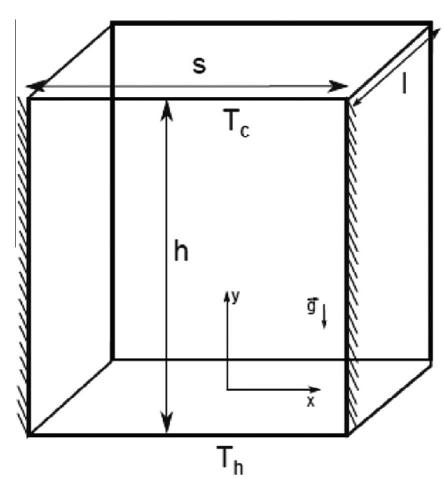

Fig. 1. Simulated cavity model with a bottom wall temperature of T_h, a top wall temperature of T_c and adiabatic side walls.