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Application of Bayesian Inference Technique for the reconstruction of an isothermal hot spot inside a circular disc from peripheral temperature measurement – A critical assessment

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ABSTRACT

In this communication an internal hot spot in a two dimensional circular domain has been reconstructed using Bayesian Inference Technique. A semi-analytical forward model based on boundary collocation method has been used for the determination of temperature distribution. The posterior mean and the maximum a posteriori estimates have been computed using the Markov Chain Monte Carlo sampling technique. Based on the problem definition the reconstruction of the hot spot requires the estimation of the location, size and boundary temperature of the spot. Estimation of each of these parameters has been done individually (single parameter estimation) as well as taking different combinations of them (multiple parameter estimation). Synthetic measurement data with and without uncertainty have been used first to critically examine the scheme of reconstruction and to judge the goodness of the scheme of estimation for each of the unknowns. An experimental scheme has also been devised to generate data for reconstruction. Estimation using data from both of these sources reveal that the uncertainty involved in the prediction of different parameters vary widely. It has been observed that the uncertainty involved in the prediction of eccentricity is much more compared to that for the prediction of the temperature or radius of the hot spot. The reason for this has been pinpointed through the sensitivity analysis. The sensitivity analysis further reveals that simultaneous estimation of all the attributes of the internal hotspot is difficult to make in the present scenario. Finally, the limitation of the present scheme has been discussed.

1. Introduction

Heat conduction problems are often encountered where all the boundary conditions for the domain of interest are specified explicitly. For such a situation, the temperature distribution (both in time and space) can be determined solving the governing equation for known thermophysical properties. Such analyses, sometimes referred to as forward or direct heat conduction problems, have an immense academic relevance and enjoy widespread applications in science and technology. There is another class of problem in which the specification of the boundary condition is only partial, however, measured temperature values (or heat flux in principle) at discrete locations are available. The purpose of solving such problems could be diverse, namely, determination of temperature in rest of the domain, estimation of unknown boundary conditions, determination of unknown thermophysical properties, identification of non-homogeneity in the domain of interest and many more. Such problems are known as inverse heat transfer problems. The solution of inverse problems not only imposes a greater challenge, sometimes unique solutions are not obtained. When the solution is not unique or unstable to the perturbations of the boundary condition or material properties, the problem is called an ill-posed problem. Nevertheless, different techniques have been evolved to address inverse heat conduction problems over the years. Many a time the ill-posed problem is converted into a conditionally well-posed one and the solution is sought through a functional optimization. Historically, the method of regularization proposed by Tikhnov [1] is a pioneering effort towards the solution of ill-posed problems. The roots of many investigations in inverse heat conduction problems (IHCP) may be found in the regularization method and its different variations. This is a well researched field as is evidenced by several reference books and review articles [2–4].

It may be noted that the experimental results, property values, process parameters are most often distorted by the noise and uncertainty in estimation. So, a need was felt to devise solution

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Nomenclature

Bi	Biot number
e	Eccentricity(m)
h	Heat transfer coefficient ($Wm^{-2}K^{-1}$)
k	Thermal conductivity ($W(mK)^{-1}$)
MAP	Maximum a posterior
$\bar{R}$	Ratio of domain and hot spot radius $= \frac{r_o}{r_i}$
R	Non-dimensional radial location $= \frac{r}{r_i}$
r	Radius(m)
R_e	Non-dimensional eccentricity $= \frac{e}{r_i}$
r_i	hot spot radius(m)
r_o	Disk radius(m)
R_s	Non-dimensional radial location on the periphery $= \frac{r_s(\theta)}{r_i}$
S	Hot spots
T	Temperature(° C)
V	Input variables or parameters for forward model

Greek Symbols

γ	Observation vector
μ_p	Mean of Gaussian prior for unknown parameters
$\boldsymbol{\Omega}$	Working domain
$\boldsymbol{\Psi}$	State vector
σ_p	Variance of Gaussian prior for unknown parameters
$\boldsymbol{\Theta}$	Non-dimensional temperature
θ	Angular position(°)

Subscripts

∞	Ambient
avg	average
ext	external
int	internal

methodologies which can take these uncertainties into cognition and can provide estimation even in the presence of such unavoidable real life aberrations. This calls for the incorporation of a stochastic or statistical approach in the solution methodology. In the last two decades there have been considerable activities in stochastic modeling of continuum systems incorporating parametric uncertainty. Several techniques [5–7] have been tried to solve inverse heat transfer problems. Out of various stochastic methods statistical framework based on Bayesian Inference Technique (BIT) has gained a wide acceptance for the solution of inverse heat conduction. While the deterministic approach mentioned in the previous paragraph seeks a single solution of the specific unknown, BIT aims at finding the posterior distribution of the unknowns in the presence of the measurement uncertainty and even the uncertainty involved in the modeling [4]. The posterior probability density function (PPDF) of the unknown parameter is obtained from a prior distribution model through the use of likelihood. The adoption of Markov Chain Monte Carlo (MCMC) sampling technique in the paradigm of BIT has extended the horizon of its application.

In a recent review Kaipio and Fox [4] has demonstrated the application of Bayesian inference method for inverse heat transfer problems (IHTP). In 2004, Wang and Zabaras [8] proposed a model to predict the boundary heat flux of a conducting solid for a given temperature field considering both 1D and 2D conduction. This inverse problem has been solved using Bayesian inference and MCMC sampling. Essentially, this basic framework has been followed for tackling inverse heat conduction problem in many subsequent investigations, though time to time some variations have been brought in the process.

It is also interesting to note that this technique has been used to estimate a variety of unknown parameters namely thermal conductivity and convective heat transfer co-efficient at the surface [9,10], time dependent heat flux [11], temperature dependent thermophysical properties and transient boundary heat flux [12], transient heat source in a radiatively participating medium [13] etc. Wang and Zabaras [13] considered a 3D combined conduction radiation problem in a participating medium with an aim to reconstruct an unknown transient heat source. They commented on the necessity of using a reduced order model to restrict the demand of computation. To this end, they have used the technique of proper orthogonal decomposition. The proposed methodology was successful in reconstructing some typical heat source profile through BIT.

Improvement on the basic BIT have been proposed by several researchers. Hierarchical Bayesian inference method [14,15] have been suggested to determine both the regularization parameter and the noise level automatically. This method quantify various uncertainties in the system and enhances the accuracy of estimation. Efforts are also made to combine BIT with artificial neural network (ANN) [16,17]. Apart from claiming several other advantages of supplementing BIT in ANN, both the investigations suggested its use particularly when the forward model is computationally too expensive. In recent years different soft computing techniques [18–22] and techniques based on Kalman Filter [23–25] have been used as stand alone procedure for IHCP. However, we refrain from elaborating such investigation as they are not directly related to the present work.

In the present investigation we propose to employ BIT for the identification of an internal hot spot. In the most generalized situation a domain of interest may have hot spots of different size and temperature. Further the situation could be transient. Determination of location, shape, size and temperature of a number of hot spots with a reasonable degree of certainty, could be a formidable task. Application of BIT for such an involved problem (particularly when the solution of the forward model requires numerical techniques) could be computationally expensive and may not be advisable. However, the reconstruction of a single internal hot spot with a lesser degree of freedom can be readily done using the BIT.

We have considered a two dimensional isotropic circular domain with a circular hot spot whose temperature, location and size are not specified explicitly. While reconstructing the hot spot, rigor of the adopted technique has been critically examined in identifying each of the unknown entities using both synthetic data and experimental results. To this end, predictions have been made taking one unknown at a time or considering a combination of them. Sensitivity analysis has been made to examine the predictability of the unknown parameters. The present work can serve as the foundation for estimation for more generalized case of multiple internal hot spots.

2. Problem definition

A generalized heat conduction problem with hot spots inside the domain has been diagrammatically represented in Fig. 1. The whole domain has been denoted by $\Omega$and the hot spots have been demarcated as $S_1, S_2, \ldots, S_m$. Assuming steady state and constant thermo-physical properties, the following may be used to determine the temperature distribution in a forward problem.

$$\nabla^2 T = 0 \text{ in } \Omega - S_1 - S_2 - \cdots - S_m \quad (1)$$

On $\Gamma_{int}(j)$,