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Chaotic natural convection in a toroidal thermosyphon with heat flux boundaries

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ABSTRACT

This computational study investigates nonlinear dynamics of unstable convection in a 3*D* toroidal shaped thermal convection loop (i.e., thermosyphon) with heat flux boundary conditions; results are compared to prior 2*D* simulations. The lower half of the thermosyphon is subjected to a positive heat flux into the system while the upper half is cooled by an equal-but-opposite heat flux out of the system. Water is employed as the working fluid with fully temperature dependent thermophysical properties and the system of governing equations is solved using a finite volume method. Numerical simulations are performed for varying magnitudes of heat flux ($1.0\ \mathrm{W/m^2} \leqslant q'' \leqslant 1.0 \times 10^4\ \mathrm{W/m^2}$) to yield Rayleigh numbers (i.e., buoyant forcing) ranging from $2.83 \times 10^4 \leqslant Ra \leqslant 2.83 \times 10^8$. Delineation of multiple convective flow regimes is achieved through evolution of the bulk-mass-flow time-series and the trajectory of the mass flow attractor. Simulation results demonstrate that multiple regimes are possible and include: (1) conduction, (2) damped, stable convection that asymptotes to steady-state, (3) unstable, Lorenz-like chaotic convection with flow reversals, and (4) high Rayleigh, aperiodic stable convection without flow reversals. For the Rayleigh numbers considered, it is observed that certain flow regimes are not accessible in toroidal simulations owing to the constraints of additional surface boundaries in a 3*D* system. The *RMS* of mass flow rate, power spectra of oscillatory behavior, dominant oscillatory frequency, and residence time are also described as a function of the buoyant forcing in the system.

1. Introduction

Natural convection and buoyancy driven dynamic systems exist over a wide range of length scales and are of notable import to the scientific, mathematic, and engineering communities. On a geophysical scale, counter-rotating convection cells within the asthenospheric layer of the Earth's upper mantle are composed of ductile rock (owing to the extremely high temperatures and pressures) and produce much of the plate tectonic behavior such as the formation of large scale ridges, trenches, and volcanic activity [1]. On a regional scale, Hadley Cells in the planetary atmosphere are intimately related to the behavior of the jet stream via Rossby waves and aid to explain large scale motions of the Earth's atmosphere as well as weather pattern dynamics as quantified by ensemble averages of teleconnection indices [2]. For example, the North Atlantic Oscillation Index (*NAO*), which pertains to the North–East US weather, is commonly used by meteorologists to quantify the oscillatory and chaotic fluctuations of the jet stream in an attempt to improve the accuracy of medium range (10–30 days) weather forecasting. On a local scale, convective thunderstorms (derechos, downbursts, and straight-line windstorms), micro-climates, and land/sea breezes are all the result of unstable, differential heating in a thermal–fluid system [3–5]. Examples of natural convection employed in engineered systems include: (1) solar water heaters, (2) nuclear reactors, (3) gas turbine blade cooling, and (4) roads and railways that pass over permafrost, among many others [6–8]. The buoyant forces resulting from thermal gradients within these fluid systems can give rise to complex mass flow circulations and aperiodic behavior.

The nonlinear dynamics of unstable convection have been studied by Lorenz [9] in his 1963 differential equation model for natural convection in Rayleigh–Bénard convection cells. This work has been studied extensively in an attempt to improve mathematical models

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Nomenclature

CCW	counter-clockwise	*Ra*	Rayleigh no.
CW	clockwise	*T*	static temperature (K)
c_p	specific heat capacity at constant pressure $\left(\frac{kJ}{kg \cdot K}\right)$	*t*	time (s)
e	specific internal energy (kJ)	$\mathbf{V}$	velocity vector (m/s)
g	gravitational acceleration (m/s^2)	α	thermal diffusivity (m^2/s)
h	convection coefficient $\left(\frac{W}{m \cdot K}\right)$	β	thermal expansion coefficient (1/K)
I	identity matrix	θ	azimuth coordinate (radians)
k	thermal conductivity $\left(\frac{W}{m \cdot K}\right)$	τ	viscous stress tensor (Pa)
L	characteristic length scale (m)	ν	kinematic viscosity (m^2/s)
$\dot{m}$	mass flow rate (kg/s)	μ	dynamic viscosity $\left(\frac{kg}{m^2 \cdot s}\right)$
p	pressure (Pa)	ρ	density (kg/m^3)
q''	heat flux $(\pm W/m^2)$		

of the earth's atmosphere. Physical and/or numerical models such as thermal convection loops, or 'thermosyphons', are a simplified geometry that represents a viable tool for studying the behavior of natural convection cells [10]. Thermosyphons are a useful construct for performing scientific studies as they limit convection to a single, large cell and thus provide the simplest physical model which allows for examination of the various flow regimes that occur in convection cells.

Thermosyphons are fluid systems in which convective flow is induced via buoyant forces that occur when sufficiently large unstable temperature gradients exist (i.e., heating from the bottom and cooling from the top). The fluid circulates within a closed, circular tube (e.g., torus) that is oriented vertically in space with the direction of gravity. The resulting thermal gradient may be accommodated by conduction, or, if the gradient is sufficiently large, buoyancy driven convection. Thermosyphons exhibit many of the typical nonlinear system dynamical effects, particularly, natural convection flow regimes wherein instabilities may grow large and significantly alter the flow behavior within the thermosyphon. The various flow regimes are typically delineated as (1) conduction and/or quasi-conduction, (2) asymptotic, stable convection with unidirectional flow, (3) unstable, Lorenz-like chaotic convection with flow reversals, and (4) high Rayleigh number, aperiodic stable, convection without flow reversals.

Comprehensive review articles written by Yang [11], Raithby and Hollands [12], and Jaluria [13] discuss several important closed-loop thermosyphon problems in various branches of engineering, geophysics, environmental sciences. The review articles [6–13] contain a wealth of literature on theoretical and experimental studies of this simple system, which exhibits typical nonlinear convective effects. Early thermosyphon studies employed 1*D* models in order to study flow behavior in a thermosyphon with the assumption that all governing parameters are uniform over any given cross section at any moment in time [14,15]. Periodic oscillations were found analytically by Keller [14] in a 1*D* model consisting of a fluid-filled tube bent into a rectangular shape and standing in a vertical plane. Gorman et al. [16] presented a quantitative comparison of the flow in a thermal convection loop with the nonlinear dynamics of the Lorenz model. Here the system was heated with constant flux over the bottom half and cooled isothermally over the top half. The boundaries of different flow regimes were determined experimentally and the characteristics of chaotic flow regimes were discussed. They also derive a relationship between the parameters of the Lorenz model and the experimental parameters of the fluid and loop. Several flow stability studies have been performed by Vijayan et al. [17] and Jiang et al. [18,19] while Desrayaud et al. [20] completed a numerical investigation of unsteady, laminar natural convection in a 2*D* convection loop maintained at a constant heat flux over the bottom half and cooled at a constant temperature over the top half. For a particular range of forcing (i.e., Rayleigh number), it has been observed that the bulk fluid motion in a thermosyphon is chaotic and undergoes flow reversals. Creveling et al. [21] proposed a positive feedback mechanism in order to explain these flow reversals in a thermosyphon.

Within the extensive body of literature pertaining to thermosyphons, only a minimal subset of studies have examined the spatiotemporal behavior of the flow-field dynamics within a thermosyphon. The thermal structure of the flow and velocity-field where characterized in time by Ridouane et al. [22,23] where they examine thermosyphons with isothermal boundary conditions in 2*D* and 3*D* geometries. It was found that for 2*D* thermosyphons, chaotic flow regimes and the associated flow reversals occur for Rayleigh numbers $9.5 \times 10^4 < Ra < 4.0 \times 10^5$. However, in the 3*D* isothermal work [23], flow reversals where not observed for Rayleigh numbers ranging from $10^3 < Ra < 2.3 \times 10^7$ with isothermal boundaries. Ridouane et al. suggest that 3*D* flow structures increase flow resistance and thus damp the flow instability mechanism responsible for bulk flow reversals observed in 2*D* loops.

The basis for exploring the heat flux boundary condition in 3*D* is driven from multiple fronts. First, the flux boundary provides a better correlation with actual laboratory experiments. And second is the fact that flow reversals were not found in 3*D* isothermal simulations [23] but are known to occur in experiments with heat flux boundary conditions. In an earlier works by Louisos et al. [24,25] a chaotic flow regime with flow reversals was found in 2*D* simulations with heat flux boundary conditions. We thus seek to extend this prior work by examining a 3*D* thermosyphon with toroidal geometry and heat flux boundaries.

The present study considers 3*D* toroidal thermosyphon simulations with iso-heat flux boundaries: heating on the bottom-half of the loop $(+q'')$ and an equal but opposite iso-heat flux cooling on the top half $(-q'')$ over the range of Rayleigh numbers from 2.83×10^4 to 2.83×10^8. Here we examine both the temporal evolution and the *RMS* value of the mass flow rate in the thermosyphon. Particular focus is placed on characterizing flow reversals as defined by the transition from clockwise (*CW*) to counter-clockwise (*CCW*) flow around the convection loop (or vice versa). The trajectory of the thermosyphon mass flow rate solution is plotted on an attractor diagram and the fixed convective equilibrium solutions are shown as 'orbital centers' for both decaying, periodic, and chaotic flow regimes. A frequency analysis is performed in order to examine the power spectra of the system and