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# Analytical model for solidification and melting in a finite PCM in steady periodic regime

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ABSTRACT

An analytical solution of the phase change problem, known as the Stefan or Moving Boundary Problem, in a PCM layer (phase change materials) subject to boundary conditions that are variable in time, is presented, in steady periodic regime. The two-phase Stefan Problem is resolved considering periodic boundary conditions of temperature or of heat flux, or even mixed conditions.

This phenomenon is present in air-conditioned buildings, the walls of which use PCM layers to reduce thermal loads and energy requirements to be compensated by the plant.

The resolution method used is one in which phasors allow the transformation of partial differential equations, describing conduction in the solid and liquid phase, into ordinary differential equations; furthermore the phasors allow transformation of the thermal balance equation at the bi-phase interface into algebraic equations. The Moving Boundary Problem is then reduced to a system of algebraic equations, the solution of which provides the position in time of the bi-phase interface and the thermal field of the layer. The solution obtained provides for different thermodynamic configurations that the layer can assume and makes the position of the bi-phase interface and the thermal field depend on the Fourier number and on the Stefan number calculated in the solid phase and in the liquid phase.

In the case of two boundary conditions represented by a single sinusoidal oscillation, a general analysis, addressed in different thermodynamic configurations obtained by varying the Fourier and Stefan number, shows the calculation procedure of the steady and of the oscillating component of the position of the bi-phase interface, of the temperature field and of the heat flux field.

In addition, we considered the particular case of a PCM layer with an oscillating temperature boundary condition on one face and a constant temperature on the other face.

The analytical procedure was also used for an analysis dedicated to the thermal behaviour of Glauber's salt subject to independent multi harmonic boundary conditions. This salt hydrate is one of the most studied, having a high latent fusion heat and a melting temperature that is suited for use in the walls of buildings.



## 1. Introduction

Among the technologies available to improve thermal performance of the walls in air-conditioned buildings, the use of PCM has attracted notable attention. These materials, due to the variability of the boundary conditions, undergo phase changes with storage of latent heat in the wall and successive releasing, and consequent modification of the thermal exchanges between the air-conditioned environment and the outdoor environment. Storage of latent heat is preferable to the storage of sensible heat due to its isothermal properties and the high energetic contribution.

In the winter heating of the environments, a part of the energy lost through an opaque element of the external envelope is used for the solid–liquid phase change of the PCM layer. This process gives rise to a storage of latent heat in the wall, which, in part, can be returned to the internal environment if the opposite liquid–solid phase change occurs subsequently. In this way, the energy lost from the environment to the outside is reduced.

In summer cooling, the presence of a PCM layer drastically reduces the solar loads entering through opaque walls; the energy stored in the wall is returned in part to the outdoor environment during nocturnal hours, prevalently following radiant exchange with the sky. The advantage is the net reduction in the loads to be removed by the air-conditioning plant, the time lag of the entering heat flux and the attenuation of temperature oscillations.

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**Nomenclature**

| | |
|---|---|
| $(a)$ | portion of the layer in phase $a$ |
| $(b)$ | portion of the layer in phase $b$ |
| $c$ | specific heat capacity [J/(kg K)] |
| $F$ | heat flux [W/m$^2$] |
| $Fo$ | Fourier number [–] |
| $H$ | latent heat of fusion [J/kg] |
| $k$ | harmonic order [–] |
| $L$ | thickness of the PCM layer [m] |
| $n$ | harmonic number [–] |
| $P$ | period of oscillation [s] |
| $Ste$ | Stefan number [–] |
| $t$ | time [s] |
| $T$ | temperature [K] |
| $t^*$ | a particular instant [s] |
| $x$ | spatial Cartesian coordinates [m] |
| $X$ | position of the bi-phase interface [m] |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | thermal diffusivity [m$^2$/s] |
| $\gamma$ | propagation constant [m$^{-1}$] |
| $\Delta t$ | finite difference time step [s] |
| $\Delta X$ | finite difference of the variation of the position of the bi-phase interface [m] |
| $\zeta$ | argument of the oscillating component of the position of bi-phase interface [rad] |
| $\vartheta$ | generic component of the temperature Fourier series expansion [K] |
| $\vartheta_p$, $\vartheta_r$ | constants of integration [K] |
| $\lambda$ | thermal conductivity [W/(m K)] |
| $\rho$ | density [kg/m$^3$] |
| $\varsigma$ | argument of the abscissa in motion [rad] |
| $\upsilon$ | velocity of the bi-phase interface [m/s] |
| $\varphi$ | argument of the oscillating component of the temperature [rad] |
| $\phi$ | generic component of the heat flux Fourier series expansion [W/m$^2$] |
| $\chi$ | generic component of the position of bi-phase interface Fourier series expansion [m] |
| $\psi$ | argument of the oscillating component of the heat flux [rad] |
| $\omega$ | angular frequency [rad/s] |

*Subscripts*

| | |
|---|---|
| 1 | face 1 |
| 2 | face 2 |
| $a$ | phase $a$ |
| $a1$ | oscillation on face 1 in a-phase |
| *anal* | analytical |
| $b$ | phase b |
| $b2$ | oscillation on face 2 in $b$-phase |
| $H$ | latent heat stored per unit time |
| $k$ | $k$th harmonic |
| $M$ | melting |
| *num* | numerical |

*Symbols*

| | |
|---|---|
| – | mean value |
| ~ | oscillating value in the time domain |
| ^ | oscillating value in the complex domain |
| \| \| | amplitude of an oscillating value |
| *arg* | argument of an oscillating value |

If the internal walls are considered, the presence of a PCM layer increases thermal storage with a consequent reduction of internal air temperature oscillations.

In order for the benefits to be continuous over time, the variability of the boundary conditions must be such as to cause a fusion cycle in the layer and successive solidification in a period of 24 h.

This paper addresses the problem of heat transfer in a PCM layer subject to phase changes due to the variability of the loadings, which act on the two faces delimiting it. The external loadings, which are variable in time, are the air temperature, solar irradiance, and infrared radiation from the sky. While internal loadings are solar radiation entering through the glazed surfaces, internal loads and the power supplied by the plant. Since the obtainable benefits in terms of energy are linked to the realisation of continuous phase change cycles, and considering that the loadings have trends that can always be expressed through the sum of periodic functions, the analysis was conducted considering the steady periodic thermal regime. This regime is representative of the thermal behaviour of the building walls, especially in summer and is used for the dynamic thermal characterisation of building walls in EN ISO 13786 [1] and in [2]. The technical Standard uses harmonic analysis in a steady periodic regime for the dynamic characterization of finite monophase layers of building components with only sensible thermal storage. The boundary conditions on the two faces delimiting the wall are temperature or heat flux that vary sinusoidally.

In a PCM layer subject to phase changes, the transfer regime of the heat flux is modified due to the discontinuity of the heat flux at the bi-phase interface due to the latent heat storage. This phenomenon determines variability in time of the position of the bi-phase interface in the layer and the modification of the thickness of both the solid phase and the liquid phase. The thermal field in the two phases, which present different thermophysical properties, is a function of the position of the bi-phase interface that is variable in time, as well as the relative boundary conditions.

From a historical survey, several authors have given exact analytical solutions only in monodimensional semi-infinite or infinite domains with simple initial and boundary conditions. However, they neglected convection in the liquid phase and the variation of the thermophysical properties in the two phases. The Stefan Problem is divided into a one-phase Stefan Problem and a two-phase Stefan Problem. The term 'one-phase' designates one of the phases being active, the other phase staying at its melting temperature, while the term 'two phase' indicates that the thermal field varies in both phases. In particular, the following one-phase problems have been solved, by using similarity variable [3–8]: (1) conduction in a semi-infinite phase change material with a constant temperature greater than zero at the initial time. In the subsequent instants a constant temperature less than zero at abscissa x = 0 causes a solidification which occurs at a temperature equal to zero; (2) conduction in an infinite phase change material with, at the initial time, a liquid phase placed in the abscissae $0 < x < +\infty$ at a temperature greater than zero and the solid phase placed in the abscissae $-\infty < x < 0$ at a temperature less than zero. Analogously to the first problem, a constant temperature less than zero at abscissa $x = 0$ causes a solidification which occurs at a temperature equal to zero. In these Stefan Problems the liquid phase stays at melting temperature during the solidification process.

The extension of the one-phase problem solution to the two-phase problem is known as Neumann's solution [3–8]. Such a Moving Boundary Problem concerns conduction in a