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Design of radiative enclosures by using topology optimization

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ABSTRACT

Thermal radiation within enclosures are widely present in applications such as furnaces, satellites and spacecrafts. Modern demands of quality and efficiency in the design of these structures require better tools than the traditional "trial-and-error" method. In that sense, topology optimization method allows a systemized distribution of material inside a design domain, such that a prescribed objective function is minimized, subjected to design constraints. It represents a robust and reliable option to design the interior of enclosures. Thus, in this work, a topology optimization design method is presented for the distribution of boundary reflectivities within diffuse-gray enclosures. The modeling of radiative heat transfer in a nonparticipating media is solved by the net-radiation method and the optimization algorithm used in this work is the method of moving asymptotes (MMA). The cases considered are the distribution of thermal radiation reflective material on flat surfaces in order to maximize or minimize the sum of net heat flux and to minimize the sum of the temperatures in a specified region of the design domain. Numerical examples are presented to illustrate the proposed design method.

1. Introduction

Thermal radiation within enclosures are widely present in applications such as furnaces, satellites and spacecrafts, and the theory that describes this phenomenon has been consistently discussed in literature [1]. The modern demands of quality and efficiency in the design of these structures require a better tool than the traditional "trial-and-error" method.

The problem of designing radiant enclosures has been extensively addressed in literature by means of inverse methods. From this approach, an inverse design problem that is solved in its ill-posed explicit form can have the ill-conditioned portion of that problem removed or improved by methods such as Monte Carlo, Thikonov method, conjugate gradient regularization, among others [2,3]. In this sense, in [4,5], the determination of temperature profiles and wall properties are considered, and in [6,7] the estimation of thermal and radiative properties of advanced materials by using regularization methods are compared with experimental data.

Currently, more attention have been paid on optimization design techniques. For this method, a design problem is implicitly defined as the minimization of an objective function subjected to some constraints and then it is iteratively solved through mathematical tools. By using optimization, the determination of heater temperatures of an industrial radiative oven such that the surface of a continuously moving load achieves a prescribed temperature profile is addressed in [8]. In [9], the steepest descent, Newton and quasi-Newton methods are applied to optimize the geometry of a 2-D radiative enclosure with diffuse walls, and in [10] optimization approaches are compared with results obtained by using regularization methods in the determination of heater settings.

Under this perspective, topology optimization represents a robust and reliable option to design the interior of enclosures, once it allows a systemized distribution of reflective material over the design domain, such that a prescribed objective function is minimized, subjected to design constraints. Widely used for structural purposes, as in the classical problem of compliance minimization [11–13], the method of topology optimization can also be applied to solve thermal problems. One of the first studies focused exclusively in thermal design of a given structure was published by [14]. In that study, a topology optimization for a heat conduction problem of minimum resistance between input and output points is presented. The application of that method can also be encountered for the design of surfaces subjected to thermal loads of convection. In that sense, Iga et al. [15] presents a work where the investigation of the influence of design-dependent effects upon heat convection is addressed. However, the bibliography that reports the application of topology optimization in radiation problems is scarce. Among the very few publications on that issue, it is possible to identify the work from [16], which considered the treatment of radiation as a nonlinear convective boundary condition in the design of micro-cooling fins.

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The topology optimization approach presented in this work makes possible the distribution of reflective material in the interior of diffuse-gray enclosures with transparent media in order to either maximize or minimize the net heat flux summation or minimize the temperature summation in a specified area of that inner domain, which is subjected to boundary conditions of prescribed temperatures and net heat fluxes. Such requirements are constantly found in the layout design of various types of modern enclosures.

This paper is organized as follows: in Section 2, the analytical governing equations of the radiative heat transfer within gray-diffuse vacuum enclosures is presented. In Section 3, the numerical solution of the problem by using the net-radiation method is described. In Section 4, the topology optimization formulation, including the definition of the objective function and its derivatives, is provided. In Section 5, numerical examples of the application of the technique are shown. Finally, in Section 6, some conclusions are inferred.

2. Governing equations

Modelling thermal behaviour of surfaces within enclosures in predominance of radiative heat transfer consists, in this work, in calculating three basic quantities: radiosity, temperature, and net heat flux. The proper determination of these measures is essential for the application of the systematical reflectivity material distribution method presented.

Among several approaches available to model radiative heat transfer within enclosures [1], the approach assumed in this work considers a vacuum enclosure, where surfaces are assumed to be isothermal, opaque, diffuse, grey and with uniform radiosity. An accurate determination of radiation under such conditions depends on the precise calculation of view factors, as noticed by [17]. Therefore, these factors, that represent the percentage of radiation that leaves surface i and reaches a surface j, are determined through the contour double integral formula [18]:

$$F_{ij} = \frac{1}{2\pi A_i}\oint_{C_i}\oint_{C_j}[\ln(r_{ij})\,dx_i dx_j + \ln(r_{ij})\,dy_i dy_j + \ln(r_{ij})\,dz_i dz_j] \tag{1}$$

where C_i and C_j are contours of boundary areas A_i and A_j, r_{ij} is the distance between two line elements on each contour and dx, dy, and dz correspond to differential lengths of those line elements.

Considering boundary conditions of prescribed temperature and net heat flux, radiosity over each surface of an enclosure can be directly calculated. Thus, for surfaces with prescribed temperature, radiosities are calculated as [1]:

$$J_i = \varepsilon_i \sigma T_i^4 + (1-\varepsilon_i)\sum_{j=1}^{N} F_{ij} J_j \tag{2}$$

where T_i is the known temperature, ε_i is the emissivity, σ is the Stefan–Boltzmann constant ($\sigma = 5.67 \times 10^{-8}$), and N is the total number of surfaces that build the enclosure. For surfaces with prescribed net heat flux, radiosities are calculated as follow [1]:

$$J_i = \frac{Q_i}{A_i} + G_i = \frac{Q_i}{A_i} + \sum_{j=1}^{N} F_{ij} J_j \tag{3}$$

where Q_i is the known net heat rate of surface i. Eq. (3) is of a particular interest in this work, once it makes possible to write net heat flux as:

$$q_i = \frac{Q_i}{A_i} = J_i - \sum_{j=1}^{N} F_{ij} J_j \tag{4}$$

Finally, unknown temperatures and net heat fluxes over surfaces can be calculated through the net-radiation method, which organizes these measures in a system of N equations, as shown below [1]:

$$\sum_{j=1}^{N}\left[\frac{\delta_{ij}}{\varepsilon_j} - F_{ij}\left(\frac{1-\varepsilon_j}{\varepsilon_j}\right)\right]\frac{Q_j}{A_j} = \sum_{j=1}^{N} F_{ij}\sigma\left(\overline{T}_i^4 - \overline{T}_j^4\right) \tag{5}$$

where δ_{ij} is the Kronecker Delta.

3. Matricial form of net-radiation method

The matricial form of net-radiation method for the calculation of radiative heat transfer in diffuse-gray vacuum enclosures follows the ideas presented in Section 2. Therefore, for an enclosure subjected to boundary conditions of temperature and net heat flux, radiosities are defined similarly as done by [19]:

$$\mathbf{K_R J} = \mathbf{E} \tag{6}$$

where $\mathbf{K_R}$ is the radiation matrix, $\mathbf{J}$ the radiosities vector and $\mathbf{E}$ the emissions vector. The essential difference between this work and [19] work consists in the approach used to assemble $\mathbf{K_R}$ and $\mathbf{E}$ which is divided in two types, as described in Section 2. Thus, for radiative surfaces with prescribed temperature:

$$K_{R_{ij}} = \frac{\delta_{ij} - (1-\varepsilon_j)F_{ij}}{\varepsilon_j} \tag{7}$$

$$E_i = \varepsilon_i \sigma \overline{T}_i^4 \tag{8}$$

and for radiative surfaces with prescribed net heat flux:

$$K_{R_{ij}} = \delta_{ij} - F_{ij} \tag{9}$$

$$E_i = q_i \tag{10}$$

A Gauss quadrature approach is adopted to numerically determine view factors F_{ij}. Over line contours that bound enclosures interior surfaces, the below formulation is applied [20]:

$$F_{ij} = \frac{1}{2\pi A_i}\sum_{el_i=1}^{N_i}\sum_{p_i=1}^{N_{gp}} W_{p_i} J_{el_i}\sum_{el_j=1}^{N_j}\sum_{p_j=1}^{N_{gp}} W_{p_j} J_{el_j}\ln\left[r(z_{p_i}, z_{p_j})\right]\hat{s}_i.\hat{s}_j \tag{11}$$

where N_i and N_j represent the amount of segments in which the contours of radiative surface i and j are divided, W_{p_i} and W_{p_j} are the weights for the numerical integration, J_{el_i} and J_{el_j} are the Jacobians calculated in the radiative surfaces el_i and el_j, r refers to the distance between the coordinates of the Gauss points z_{p_i} and z_{p_j} and $\hat{s}_i.\hat{s}_j$ is the scalar product between the versors that guide the parameterization of the contours C_i and C_j. The view factor value F_{ij} is organized into a matrix $\mathbf{F}$, so that the sum of its lines is approximately one.

After the determination of radiosities, it is possible to calculate net heat fluxes as suggested by Eq. (4):

$$\mathbf{q} = \mathbf{J} - \mathbf{FJ} \quad \text{or} \quad \mathbf{q} = (\mathbf{I} - \mathbf{F})\mathbf{J} \tag{12}$$

where $\mathbf{I}$ is an identity matrix. For determination of unknown net heat fluxes and temperatures, the net-radiation method can be defined in a matricial form as [21]:

$$\{(\mathbf{I}-\mathbf{F})[\mathbf{I} - (\mathbf{I} - diag(\boldsymbol{\varepsilon}))\mathbf{F}]^{-1} diag(\boldsymbol{\varepsilon})\sigma\}\hat{\mathbf{T}} = \mathbf{q} \tag{13}$$

where $diag(\boldsymbol{\varepsilon})$ is a diagonal matrix for emissivities of inner structures, $\mathbf{q}$ is a vector of net heat fluxes and $\hat{\mathbf{T}}$ is given by:

$$\hat{\mathbf{T}} = \begin{bmatrix}\hat{T}_1\\ \hat{T}_2\\ \vdots\\ \hat{T}_N\end{bmatrix} = \begin{bmatrix}\overline{T}_1^4\\ \overline{T}_2^4\\ \vdots\\ \overline{T}_N^4\end{bmatrix} \quad \text{and} \quad \overline{\mathbf{T}} = \begin{bmatrix}\overline{T}_1\\ \overline{T}_2\\ \vdots\\ \overline{T}_N\end{bmatrix} = \begin{bmatrix}\hat{T}_1^{\frac{1}{4}}\\ \hat{T}_2^{\frac{1}{4}}\\ \vdots\\ \hat{T}_N^{\frac{1}{4}}\end{bmatrix} \tag{14}$$

Eq. (13) can be rewritten as follow: