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A finite element method for prediction of unknown boundary conditions in two-dimensional steady-state heat conduction problems

Worasit Kanjanakijkasem

Deparment of Mechanical Engineering, Burapha University, 169 Long-Hard Bangsaen Rd., Saen Sook District, Amphur Mueang, Chonburi 20131, Thailand

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ABSTRACT

This paper presents a finite element method in predicting unknown boundary conditions of homogeneous and two-layered materials subjected to steady-state heat conduction. Firstly, a finite element formulation is introduced to solve a steady-state boundary inverse heat conduction problem of homogeneous material. Effects of bias error of temperature specified on interior nodes and locations of such nodes on accuracy of predicted temperature distribution are examined. Then, modified cubic spline is specified on material boundary to stabilize predicted temperature distribution. Cubic spline functions using different numbers of control points are used in examining their effects on accuracy of predicted temperature distribution and computing time when specifying no bias temperatures. Finally, the formulation with cubic spline function specification is employed in predicting unknown boundary conditions of two-layered materials with thermal conductivity ratio of 0.1, 1, and 10. Concept of coincident nodes is applied in handling physical condition characterized by thermal contact resistance and heat source strength at layer interface. Effect of bias error of temperatures specified on nodes within thicker layer is examined under three interface conditions. Cubic spline function with five control points can predict temperature distributions accurately for all interface conditions when specifying no bias temperatures. RMS errors vary linearly with bias errors for interface conditions with no heat source but are drastically affected by bias error when heat source exists at the interface.

1. Introduction

One category of heat conduction problem called boundary inverse heat conduction problem (IHCP) arises in many engineering contexts when surface temperature and surface heat flux of conducting solid cannot be directly measured due to inaccessibility of sensors [1], severe thermal environment on its surface [2], or avoiding intrusive measurement on its surface [3]. Mathematically, boundary inverse heat conduction problem is ill-posed and its solutions become unstable due to measurement errors [4]. Several techniques have been developed to obtain stable solutions of inverse heat conduction problems after the pioneer work of Stolz [5] but powerful technique commonly used is regularization technique developed by Tikhonov [6] including iterative regularization [7] and function specification [8].

Investigation on multidimensional inverse heat conduction problems of homogeneous material has been significantly increased since the mid of 1980s [9–17]. Among of numerous works, a number of steady-state problems have been studied [18–21]. On the contrary, solutions of inverse heat conduction problems of multi-layered materials are rare even for steady-state problems although a large number of solutions of direct problems can be found [22–26]. This is because dealing with such problems is much more difficult due to physical condition at layer interfaces [27]. Literatures on inverse heat conduction problems of multi-layered material recently appear are Wei and Li [28] and Movahedian and Boroomand [29].

The objective of this paper is to present a finite element formulation in predicting unknown boundary conditions of homogeneous and two-layered materials subjected to steady-state heat conduction. Firstly, a finite element formulation is introduced and examined by solving boundary inverse heat conduction problems of homogeneous material. Then, it is incorporated with modified cubic spline function specification [30] to stabilize predicted temperature distribution on material boundary. Finally, the formulation with cubic spline function specification is applied in predicting unknown temperature distribution on boundary of two-layered material with the aid of coincident nodes [31] for thermal conductivity ratio of 0.1, 1, and 10 under three conditions at layer interface.

E-mail addresses: marveric111@yahoo.com, worasit@buu.ac.th

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Nomenclature

a, b	side lengths of rectangular element in x-, y-direction, m
$\mathbf{b}$	modified vector of thermal loads
k	thermal conductivity, W/(m K)
$\mathbf{r}$	vector of residuals
s	coordinate along domain boundary, m
t	element thickness, m
x, y	global coordinates, m
$\mathbf{x}$	vector of unknowns
A	area of two-dimensional domain, m^2
$\mathbf{A}$	modified coefficient matrix
$\mathbf{K}$	coefficient matrix, W/K
N	element interpolation function
$\dot{Q}$	heat transfer rate, W
$\dot{\mathbf{Q}}$	vector of heat transfer rates, W
R	thermal contact resistance, m^2 K/W
S	planar heat source strength, W/m^2
T	temperature, K
$\mathbf{T}$	vector of temperatures, K

Greek symbols

ε	relative error
η	thickness of upper layer of two-layered material, m
ξ, η	local coordinates, m

Subscripts

1, 2	layer number
in	into the domain
out	out of the domain
x, y	component in x-, y-direction

Superscripts

0	initial guess
e	element
i	equation number in system of equations
k	iteration number
s	system of equations for whole domain

2. Analytical solutions of steady-state heat conduction problems

In this section, analytical solutions of steady-state heat conduction in homogeneous and two-layered materials on rectangular domain are derived for constructing of steady-state boundary inverse heat conduction problems in remaining sections.

2.1. Homogeneous material

Governing equation of steady-state heat conduction without volumetric heat generation in an isotropic homogeneous rectangular plate with constant properties shown in Fig. 1 is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{1}$$

The method of separation of variables yields analytical solution in the form

$$T(x,y) = \sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{l_x} \sinh\frac{n\pi y}{l_x} \tag{2}$$

where A_n's are constant determined from boundary condition at $y = l_y$.

For $f(x) = T_0 \sin\frac{\pi x}{l_x}$, $A_n \neq 0$ when $n = 1$ and

$$A_1 = \frac{T_0}{\sinh\frac{\pi l_y}{l_x}} \tag{3}$$

Therefore, analytical solution for this condition is

$$T(x,y) = T_0 \frac{\sin\frac{\pi x}{l_x} \sinh\frac{\pi y}{l_x}}{\sinh\frac{\pi l_y}{l_x}} \tag{4}$$

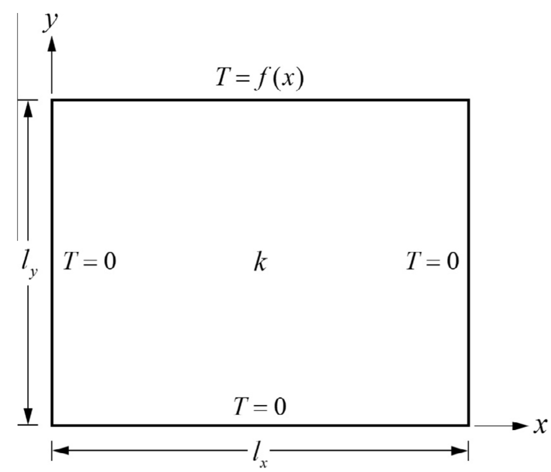

Fig. 1. Steady-state heat conduction in homogeneous material.

2.2. Two-layered material

Assuming that there is no volumetric heat generation within two-layered material and both layers are isotropic homogeneous materials with constant properties, governing equations for steady-state heat conduction of two-layered material shown in Fig. 2 are

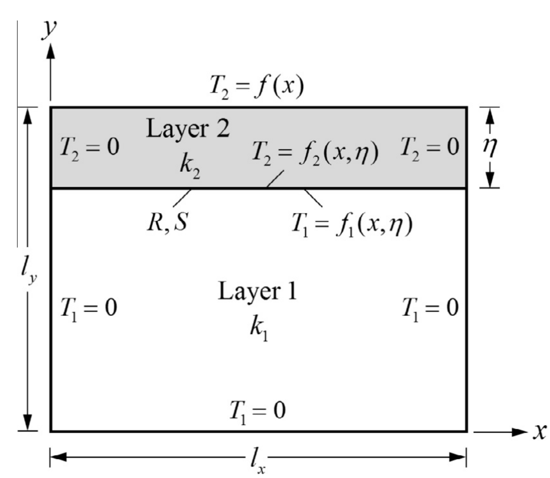

Fig. 2. Steady-state heat conduction in two-layered material.