International Journal of Heat and Mass Transfer 87 (2015) 36–41

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Free convection heat transfer in a square cavity filled with a porous medium saturated by a nanofluid

T. Groşan [a], C. Revnic [b], I. Pop [a,*], D.B. Ingham [c]

[a] Department of Mathematics, Babeş-Bolyai University, 400084 Cluj-Napoca, Romania
[b] Faculty of Pharmacy, University of Medicine and Pharmacy, Cluj-Napoca, Romania
[c] Centre for Computational Fluid Dynamics, University of Leeds, Leeds LS2 9JT, UK

ARTICLE INFO

Article history:
Received 2 February 2015
Received in revised form 19 March 2015
Accepted 23 March 2015
Available online 10 April 2015

Keywords:
Nanofluid
Porous medium
Square cavity
Heat generation
Numerical results

ABSTRACT

The steady natural convection in a two-dimensional porous square cavity filled with a nanofluid and including internal heat generation is presented in this paper. The mathematical model considered consists of the Darcy equation for the momentum, while a new model is proposed for the energy and nanoparticles' concentration equations. The system of partial differential equations is written in terms of a dimensionless stream function, temperature and concentration of the nanoparticles and is solved numerically using the finite difference method. The effects of the governing parameters, such as: Rayleigh number Ra, the Lewis number Le and the dimensionless ratio between the thermophoretic and Brownian coefficients N_{BT} on the velocity, temperature and nanoparticles' concentration, Nusselt and Sherwood numbers are studied. It is found that the addition of the nanoparticles into the fluid saturated porous medium reduces the temperature and enhances the heat transfer when the value of the thermal conductivity of the nanoparticles is much higher than the thermal conductivity of the solid structure of the porous medium. This addition has almost no effect on the flow and heat transfer characteristics when the values of the thermal conductivity of the nanoparticles and the thermal conductivity of the solid structure of the porous medium have close values.

1. Introduction

Due to the multitude of applications in thermal and geothermal energy, porous catalysts, soil pollution, nuclear reactors cooling, combustion technology, fuel cells, etc., convective heat transfer in enclosures filled by saturated porous media with internal heat generation continues to be extensively investigated. Comprehensive physical and mathematical aspects of convection in porous media are widely presented in the classical books by Nield and Bejan [1], Ingham and Pop [2], Vafai [3], and Vadasz [4].

Over the last twenty years, or so, a new hot topic, namely nanofluids, appeared in the field of heat transfer. Many industrial processes, biology, medicine, catalytic chemistry end environmental applications started to use nanotechnologies (see [5,6]). Different mathematical models have been employed by several authors to describe heat transfer in nanofluids. Among all these models, the most used are those where the concentration of the nanoparticles is constant and the addition of the nanoparticles into the base fluid improved their physical properties [7,8]. Moreover, other models that are based on the variation of the physical properties, include thermal dispersion [9,10] or Brownian motion [11]. A more complex mathematical model [12] considers that the concentration of the nanoparticles is variable and incorporates the effects of Brownian motion and thermophoresis. Recently, Celli [13] had the idea to combine the model proposed by Buongiorno [12] and a model based on improved physical properties and considering that the last one is an average concentration of the nanoparticles.

However, there are few studies regarding convective flow in enclosures filled with a nanofluid saturated porous media. Mathematical models used in these studies are based directly on the model used by Tiwari and Das [7] (Sheikholeslami et al. [14], Chamkha and Ismael [15], Akbar et al. [16]) or model [12] (Nield and Kuznetsov [17], Sheremet and Pop [18], Sheremet and al. [19]) Recently, Sheremet et al. [20] have studied the convective heat transfer in a differentially heated cavity filled by a nanofluid saturated porous medium using a new model based on the Tiwari and Das [7] model, but adapted to nanofluid saturated porous media. Other studies in 2D and 3D cavities filled with nanofluid saturated porous media using the nanofluid model proposed by Buongiorno [12] are, for example, Sheremet and Pop [21,22],

* Corresponding author.

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2015.03.078
0017-9310/

Nomenclature

D_B	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
g	gravitational acceleration
k	thermal conductivity
K	permeability of the porous medium
L	length of the cavity
$m_{w,0}$	mass transfer from hot wall
Nb	Brownian motion parameter
Nr	buoyancy-ratio parameter
Nt	thermophoresis parameter
Nu	local Nusselt number
$\overline{Nu}$	mean Nusselt number
$q_{w,0}$	heat flux from the hot wall
Ra	Rayleigh number for the porous medium
Sh	Sherwood number
T	temperature of the fluid
T_h	temperature of the hot wall
T_c	temperature of the cooled wall
u v	dimensional velocity components along the axes $\bar{x}$, $\bar{y}$
U, V	dimensionless velocity components along the axes X, Y
x, y	dimensional Cartesian coordinates
X, Y	dimensionless Cartesian coordinates

Greek letters

α_m	thermal diffusivity of the porous medium
β	volumetric expansion coefficient of the fluid
ε	permeability of the porous medium
ϕ	nanoparticle volume fraction
ϕ_0	initial nanoparticle volume fraction
Φ	rescaled nanoparticle volume fraction
θ	dimensionless temperature
μ	dynamic viscosity
ρ_f	fluid density
ρ_p	nanoparticle mass density
$(\rho C_p)_f$	heat capacity of the fluid
$(\rho C_p)_p$	effective heat capacity of the nanoparticle material
σ	$= (\rho C_p)_f/(\rho C_p)_p$, Eq. (8)
τ	$= \alpha_{mnf} t/\sigma L^2$, dimensionless time, Eq. (8)
Ψ	dimensionless stream function

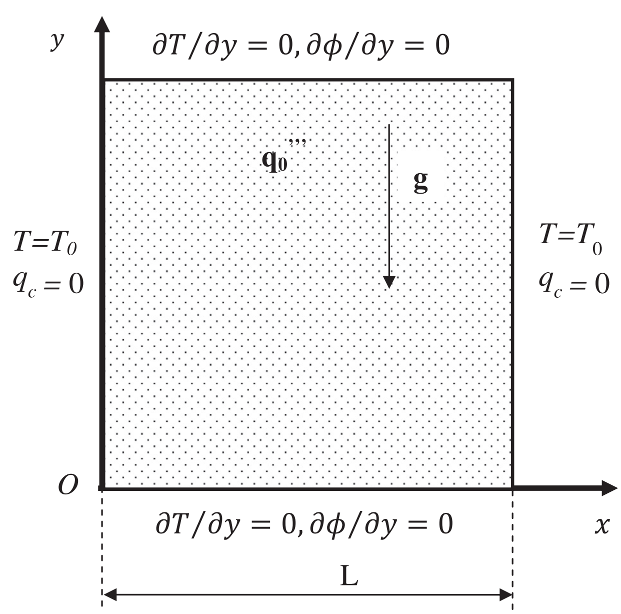

Fig. 1. Physical model and coordinate system.

Elahhi et al. [23], Akbar et al. [24,25], Sheikholeslami et al. [26,27] and Rashidi et al. [28].

In the present paper we propose a new mathematical model for free convection in a nanofluid saturated porous media by expanding the Buongiorno's model [12] by considering the effect of the porous medium and nanoparticles physical properties. Using this new model, the convective heat transfer in a square cavity filled with a nanofluid saturated porous medium in the presence of internal heat generation is studied.

2. Basic equations

Consider the free convection in a two-dimensional porous square cavity filled with a nanofluid based on water and different types of nanoparticles: A schematic geometry of the problem under investigation is shown in Fig. 1, where x and y are the Cartesian coordinates and L is the height of the cavity. It is assumed that the vertical walls are maintained at a temperature T_0 and that the nanoparticles flux $\mathbf{q_c} = D_B\nabla\phi + (D_T/T_0)\nabla T$ is zero on the solid walls while the horizontal walls are adiabatic $(\partial T/\partial y = 0)$. Here, T is the fluid temperature, ϕ is the nanoparticle volume fraction, D_B is the Brownian diffusion coefficient and D_T is the thermophoretic diffusion coefficient. Using the Darcy–Boussinesq approximation, and following the nanofluid model proposed by Buongiorno [12], the basic equations are given by (see [1]):

$$\nabla \cdot \mathbf{v} = 0 \tag{1}$$

$$0 = -\nabla p - \frac{\mu_{mnf}}{K}\mathbf{v} + (\rho\beta)_{nf}(T - T_0)\mathbf{g} \tag{2}$$

$$(\rho c)_{mnf}\frac{\partial T}{\partial t} + (\rho c)_{nf}(\mathbf{v}\cdot\nabla)T = k_{mnf}\nabla^2 T + \varepsilon(\rho c)_p\left(D_B\nabla\phi + \frac{D_T}{T_0}\nabla T\right)\cdot\nabla T + q_0''' \tag{3}$$

$$\frac{\partial\phi}{\partial t} + \frac{1}{\varepsilon}(\mathbf{v}\cdot\nabla)\phi = \nabla\left(D_B\nabla\phi + \frac{D_T}{T_0}\nabla T\right) \tag{4}$$

along with the boundary conditions:

$$\begin{aligned} &T = T_0, \quad D_B\frac{\partial\phi}{\partial y} + \frac{D_T}{T_0}\frac{\partial T}{\partial y} = 0 \quad \text{at} \quad x = 0, L \\ &\frac{\partial\phi}{\partial y} = 0, \quad \frac{\partial T}{\partial y} = 0 \quad \text{at} \quad y = 0, L \end{aligned} \tag{5}$$

where ∇^2 is the Laplace operator, $\mathbf{v}$ is the Darcian velocity, T is the nanofluid temperature, ϕ is the nanoparticle volume fraction, K is the permeability of the porous medium, and ε is the porosity. The physical properties of the nanofluid (viscosity, heat capacitance, thermal conductivity, buoyancy coefficient) are given by [9]:

$$\begin{aligned} &\mu_{nf} = \frac{\mu_f}{(1-\phi_0)^{2.5}}, \quad (\rho C)_{nf} = (1-\phi_0)(\rho C)_f + \phi_0(\rho C)_p, \\ &\frac{k_{nf}}{k_f} = \frac{(k_p+2k_f) - 2\phi_0(k_f-k_p)}{(k_p+2k_f) + \phi_0(k_f-k_p)}, \quad (\rho\beta)_{nf} = \phi_0(\rho\beta)_p + (1-\phi_0)(\rho\beta)_f \end{aligned} \tag{6}$$

where ϕ_0 is the initial uniform concentration of the nanoparticles in the cavity and nf, f and p refer to the nanofluid, fluid and (nano)-particle, respectively. On the other hand, the nanofluid saturated