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Modeling of the experiments on the Marangoni convection in liquid bridges in weightlessness for a wide range of aspect ratios

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ABSTRACT

Hydrodynamic stability of a two-dimensional steady thermocapillary flow under weightlessness in a high-Prandtl number liquid bridge is studied by means of three-dimensional numerical modeling for a wide range of aspect ratios. We suggest an explanation of the findings of a series of microgravity experiments on Marangoni convection in liquid bridges. Stability of the flow with heat transfer through the interface, modeled by the classical Fourier law, is compared with the stability of the same system under adiabatic conditions. Cooling the interface may significantly shift the threshold of hydrothermal instability as soon as the Biot number deviates from zero. It may also affect the structure of the basic Marangoni flow and the mode of the supercritical flow. We demonstrate that the heat loss has a destabilizing effect for the aspect ratios (ratio of radius to height) below 2.4 (with the exception of a region between 1.6 and 1.8), and for the longer liquid bridges the prevailing effect is stabilizing. The heat transfer coefficient as a function of the length of the liquid zone is theoretically calculated using a model of heat transport for laminar forced convection. Comparison of the results of the modeling with the experimental data shows that an incorrect assessment of the heat transfer may lead to wrong conclusions concerning both the critical parameters of the flow and its structure.

1. Introduction

A surface tension gradient induces tangential stresses, which generate a large-scale interfacial motion, the so-called Marangoni flow. A flow associated with an inhomogeneity of temperature is called thermocapillary. These flows are omnipresent both in nature and in many industrial applications, such as crystal growth, evaporation and welding. In a microgravity environment, Marangoni flows play a dominant role in heat and mass transfer and become very important for many space applications.

The geometry of interest in the present study is a non-isothermal cylindrical column, called liquid bridge (we will refer to also as *LB*), which is a droplet of a liquid confined between two differentially heated parallel flat disks. Liquid bridge is a model representing half of a floating zone – a technological process of crystal growth [1]. The thermocapillary flow is directed from the hot boundary as for many fluids surface tension is a decreasing function of temperature.

The steady two-dimensional toroidal flow is the stable mode of convection at small values of the imposed temperature difference ΔT between the disks. For any chosen geometry and parameter space, there exists a critical value of temperature difference ΔT_{cr}, above which an instability sets in [2] and gives rise to a number of time-dependent three-dimensional flow regimes. In particular, it may generate standing or traveling hydrothermal waves or lead to temporally chaotic dynamics [3–5]. A traveling wave can propagate in the azimuthal [6–8] or axial [9–11] direction and be characterized by a single integer azimuthal wave mode m or a combination thereof [12]. Varying the applied temperature difference and the properties of the liquid, most importantly the Prandtl number ($Pr = \nu/k$) [13,14] (defined as the ratio between the kinematic viscosity ν and the thermal diffusivity k of the fluid), one can study a variety of dynamical regimes.

Much efforts, both theoretical [15,16] and experimental [2,17–19] were put into understanding the reason for the onset of instability and measuring the critical parameters. The critical temperature difference, or suitably defined the critical *thermocapillary* Reynolds number $Re_{cr} \propto \Delta T_{cr}$, and the wave number m were calculated and measured for different liquids and shapes of non-cylindrical interface. The first empirical correlation for the determination of the azimuthal wave number realized near the critical point $m \approx \frac{2.2}{\Gamma}$ (Γ is the height to radius of the liquid bridge ratio), has been suggested by Preisser et al. [2]. The same

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relation but with slightly different coefficient 2.0 has been obtained numerically for $Pr < 7$ assuming pure Marangoni convection [14]. Varying buoyancy forces results in a different value of the coefficient [7].

Experimental evidences of the important role of the heat transport through the liquid–gas interface in the stability of the thermocapillary flow in LB are being reported since the 1980s. In recent years there has been much progress in the understanding of the role of thermal conditions at liquid–gas interface in hydrodynamic stability of thermocapillary flows. Previous experimental and theoretical works, [18–26], have demonstrated that varying thermal conditions at a liquid–gas interface (the rate of heat exchange and ambient gas temperature) changes significantly both ΔT_{cr} and structure of supercritical thermocapillary flow. Heat flux through interface may provoke a change of mode m of the flow [25,26].

A series of microgravity experiments on the thermocapillary convection in liquid bridges, called Marangoni Experiment in Space (MEIS) [27,28] is being performed on board the International Space Station. Among its main objectives were to measure critical parameters, and to study their dependence on thermal and kinematic conditions in surroundings.

The objectives of this work is to model the *MEIS* experiments performed with 5 cSt silicone oil with $Pr = 67$. To this end we study the onset of hydrothermal instability of thermocapillary flow by direct numerical simulations. For a liquid with such a large value of *Pr*, the anticipated mode of instability is oscillatory. Attempting to reproduce the experimental observations, we show how important it is to *correctly* take into account heat transfer between liquid and surrounding gas phases. Though the heat transfer is modeled by a simplified theoretical approach, some insight has been gained into its effect on the onset of instability.

2. Mathematical formulation

2.1. Governing equations and boundary conditions

We consider a differentialy heated cylindrical liquid column held between two flat concentric disks, as sketched in Fig. 1. The problem is examined under zero-gravity conditions with non-compressible Newtonian fluid. The liquid bridge is of radius $R = 15$ mm and of variable height d. The liquid volume is equal to that of the cylinder $\pi R^2 d$, therefore no deformation of the interface is anticipated. The liquid volume is 5% smaller than that of cylinder in the MEIS experiment.

The temperatures T_h and T_c $(T_h > T_c)$ are prescribed at the upper and lower walls respectively thereby yielding a temperature difference $\Delta T = T_h - T_c$. Density ρ, surface tension σ, and kinematic viscosity ν of the liquid are taken as linear functions of the temperature:

$$\rho(T) = \rho_0 - \rho_0\beta(T - T_0), \quad \beta = -\frac{1}{\rho_0}\frac{\partial\rho}{\partial T},$$
$$\sigma(T) = \sigma_0 - \sigma_T(T - T_0), \quad \sigma_T = -\frac{\partial\sigma}{\partial T},$$
$$\nu(T) = \nu_0 + \nu_T(T - T_0), \quad \nu_T = \frac{\partial\nu}{\partial T}.$$

where the reference temperature $T_0 = T_c$. Therefore, hereafter the subscript index "0" denotes the value of a parameter at T_c.

The governing dimensionless Navier–Stokes, continuity and energy equations for an incompressible fluid are:

$$\frac{\partial\mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V} = -\nabla P + R_\nu\cdot 2\mathbf{S}\times\nabla\Theta + (1 + R_\nu\Theta)\nabla^2\mathbf{V}, \quad (1)$$
$$\nabla\cdot\mathbf{V} = 0, \quad (2)$$
$$\frac{\partial\Theta}{\partial t} + \mathbf{V}\cdot\nabla\Theta = \frac{1}{Pr}\nabla^2\Theta, \quad (3)$$

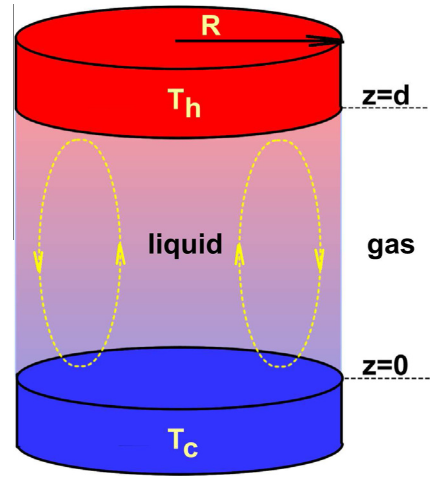

Fig. 1. Liquid bridge of radius R and height d with a straight interface. The dashed closed lines with arrows schematically show the thermocapillary and return flows (called basic Marangoni flow) and their directions in the bulk.

where $\mathbf{V} = (V_r, V_\varphi, V_z)$ is velocity, $\Theta = (T - T_0)/\Delta T$ is temperature and t is time, $\mathbf{S}$ is the strain rate tensor. The scales for time, velocity and pressure are $t_{ch} = d^2/\nu_0$, $V_{ch} = \nu_0/d$ and $P_{ch} = \rho_0 V_{ch}^2$. ∇ is the nabla operator. The scales for the radial and axial coordinates are the radius R and the height d of the liquid column, respectively.

The governing equations are solved with the following boundary conditions. At the rigid walls, no-slip and impermeability conditions are imposed:

$$\vec{\mathbf{V}}(r,\varphi,z=0,t) = \vec{\mathbf{V}}(r,\varphi,z=1,t) = 0,$$
$$\Theta(r,\varphi,z=0,t) = 0, \quad \Theta(r,\varphi,z=1,t) = 1.$$

On the free surface at $r = 1$:

$$V_r = 0, \quad 2[1 + R_\nu\Theta]\mathbf{S}\cdot\mathbf{e_r} + Re\left(\mathbf{e_z}\partial_z + \mathbf{e}_\varphi\frac{1}{r}\partial_\varphi\right)\Theta = 0, \quad (4)$$
$$\partial\Theta/\partial r = -Bi(\Theta_s - \Theta_{amb}), \quad Bi = \frac{R}{\lambda_l}h, \quad (5)$$

where *Bi* is the Biot number, $\Theta_s = (T_s - T_c)/\Delta T$ is the dimensionless temperature at the interface, and $\Theta_{amb} = (T_{amb} - T_c)/\Delta T$ is the dimensionless temperature of gas phase near the interface (T_s and T_{amb} are the dimensional temperatures of liquid at the interface and of gas), h is the heat transfer coefficient, λ_l is the thermal conductivity of the liquid. As seen from Eq. (5), the heat exchange between the liquid and the gas is controlled by both the Biot number and the temperature profile in the gas.

The Biot number is a dimensionless parameter whose value depends not only on the physical properties of the media but also on features of the flow. Calculation of the heat transfer coefficient and the choice of both T_{amb} and *Bi* will be discussed in Section 4.

Besides the Biot number and Θ_{amb}, the problem is completely described by four non-dimensional parameters - the *thermocapillary* Reynolds number, *Re*, the viscosity contrast, R_ν, the Prandtl number, and the aspect ratio Γ:

$$Re = \frac{\sigma_T\Delta T d}{\rho_0\nu_0^2}, \quad R_\nu = \frac{\nu_T\Delta T}{\nu_0}, \quad Pr = \frac{\nu_0}{k_0}, \quad \Gamma = \frac{d}{R},$$

where k is the thermal diffusivity.

The working fluid is 5 cSt silicone oil, which is a viscous weakly evaporating substance. Its physical properties are listed in Table 1.