Joule heating induced interfacial instabilities in free-surface electro-osmotic flows

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ABSTRACT

Application of an external electrostatic field across a conducting electrolyte results in the development of a thermal gradient within the fluid domain as a consequence of the heat generated due to Joule heating. The generalized Gouy–Chapman theory for symmetric electrolytes is used to explain the explicit dependence of the electrical potential and velocity distribution on this differential temperature distribution within the fluid. Subsequently the highly coupled system of Orr–Sommerfeld equations is solved using a Chebyshev collocation algorithm to perform a linear stability analysis of the free-surface electro-osmotic flow system of a conducting electrolyte. An extensive parametric study, consisted with the electrokinetic and thermal properties of the electrolyte as well as the applied field strength characteristics, is performed. In particular, the role of the two important dimensionless parameters representing the applied field strength and the electrical conductivity of the fluid in regulating the length and time scales of instability occurring at the free liquid–air interface is summarized in this study, as well as their combined influence on the inception and growth of the interfacial mode of instability.

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1. Introduction

Widespread applications of microfluidic devices in fields ranging from transport, mixing, separation of samples in biological and chemical analysis [1,2] to electronic circuits and micro electro-mechanical systems (MEMS) have attracted considerable interest in recent years [3,4]. The advantages of portability, increased efficiency and throughout with reduced time and reagent cost, and precise control of such small volume of liquids without any moving parts have been some of the very important factors driving the upsurge of interest in this field. Liquids in microfluidic devices can be transported liquids by electro-osmosis and particles by electrophoresis as a result of the interaction of the applied electric field with the electrical double layer formed at the solid–liquid interface [5–7].

The basic mechanism and characteristics of EOF in closed micro-channels have been thoroughly studied and reviewed [8,9]. However, the field of free-surface EOF and interfacial deformations of such surfaces has emerged to the forefront only in the last few years with potential applications in next generation MEMS devices for mixing, cleaning, heat and mass transfer processes. Some of the pioneering studies on free-surface EOF

[10–12] reported the occurrence of a new kind of hydrodynamic instability when a liquid with a free surface is subjected to electro-osmotic forces. It was also shown that the all open channel EOFs are unstable regardless of the applied field strength, and the development of these surface corrugations were further studied in detail [13]. Choi et al. [14] further generalized the study by taking into account the full description of the Maxwell stress tensor in the momentum equations, and showed that a free surface can actively participate in the flow driving mechanism and can generate counter-intuitive flow behaviors, such as flow reversal adjacent to the free surface. Mayur et al. [15,16] performed a long-wave asymptotic Orr–Sommerfeld analysis of the free surface EOF to highlight the role of the Maxwell stresses on the stability of such thin films taking into account all the destabilizing factors, including the Coulombic and van der Waals force with the stabilizing capillary forces. A detailed parametric analysis of the flow profiles of such a system was later performed [17], which showed that the flow-controlling parameters consist of not only the electrochemical properties of the electrolyte and the bounding domain, but also the geometry of the channel and its aspect ratios. Through a systematic Orr–Sommerfeld as well as long-wave analysis Ray et al. [18,19] showed the existence of two different modes of instability in the free surface EOF's. Along with the long-wave interfacial mode of instability [20], they showed the presence of a finite-wavenumber shear flow mode, caused by the strong

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applied fields for sufficiently high flow rates. A non-linear analysis of this long-wave interfacial mode [21] was further performed in the coupled EOF and EPF situations to understand the spatiotemporal evolution of these surface deformations.

Given that the main objective of this electro-osmotic flow is to generate a pressure difference for pumping the fluid through the micro-channel, high external electric fields are usually applied across the channels. An inevitable outcome of these applied potential gradients is the heat generation in the bulk as a result of volumetric Joule heating [22,23], which can lead to an overall increase in the temperature of the liquid. On one hand this temperature increase may lead to denaturation of biological samples such as proteins, DNA [24] and malfunctioning of chip systems and bubble formation [25], while on the other hand this additional thermal energy can be effectively harnessed for related temperature gradient driven processes such as temperature gradient focusing [26–28], electro-thermally induced fluid flow [29,30] and PCR reactors [31]. Joule heating causes both axial [32–35] and lateral [36,37] temperature gradients inside the micro-channels subjected to external electric field. The temperature variations cause significant changes in the temperature dependent physical properties of the fluid such as fluid viscosity, electrical and thermal conductivity, dielectric permittivity, ion concentration and surface tension [38–40]. The effects of the Joule heating, however, have not reported before in association with the free-surface EOF. Since the corrugation of the free surface generated by the inherent instability can further alter the electric current driving the Joule heating, significant hydrothermal interactions can be anticipated due to the presence of the free surface.

In this study we perform for the first time a linear stability analysis of the free-surface EOF system, to which the Joule heating effects have been added. The effect of heat transfer on the isothermal free-surface EOF is thus analyzed, showing in particular the effects of electrical conductivity on the length and time scales of the instabilities at the liquid–air interface. The rest of the paper is organized as follows. In Section 2, we present the theoretical formulation for the system followed by the base state analysis in Section 3. In Section 4, results of significant interest are discussed followed by concluding remarks in Section 5.

2. Problem formulation

2.1. System description

We consider an electrolyte layer of thickness d resting on a thermally non-conductive substrate subjected to an externally applied electric field E_a in the lateral direction parallel to the substrate, as shown in Fig. 1, where x and y represent the horizontal and vertical coordinate, respectively. The electrolyte layer is assumed to be composed of Newtonian fluid with density ρ, viscosity μ, electrical conductivity σ, dielectric permittivity ε and kinematic viscosity $v(= \mu/\rho)$. Its top is in contact with an inert gas of temperature T_g, and thus is free to deform. The zeta potentials on the substrate (solid–electrolyte interface) and the free surface (electrolyte–gas interface) are ζ_S and ζ_I, respectively.

2.2. Model for electric field

The electric field potential $\varphi(x, y)$ for the electro-kinetic flow has combined contributions from the electrical potential φ_a due to the externally applied electric field in the x-direction as well as the induced potential φ_i due to the zeta potentials in the y-direction.

$$\varphi = \varphi_a + \varphi_i. \tag{1}$$

For the present work, we assume constant surface potentials and zero induced charge at the liquid–air interface. The externally applied electric field E_a is governed by the following conservation laws

$$\nabla \bullet E_a = 0, \tag{2}$$

$$\nabla \times E_a = 0. \tag{3}$$

Here $E_a = -\nabla \varphi_a$ is the applied electric field, which can be expressed as the gradient of the potential. Substituting the potential function in Eqs. (2) and (3), leads to the Laplace equation,

$$\nabla^2 \varphi_a = 0. \tag{4}$$

The applied electric field decays linearly across the electrodes, giving $\frac{\partial \varphi_a}{\partial x} = -E_a$. Also at $x = 0$, $\varphi_a = 0$. Thus, solving the above Laplace equation with these two boundary conditions gives

$$\varphi_a = -E_a x. \tag{5}$$

The potential φ_i due to zeta potential is governed by the Poisson equation given by

$$\nabla \cdot (\varepsilon \nabla \varphi_i) = -\rho_e. \tag{6}$$

Here ρ_e is the net charge density and is defined as

$$\rho_e = zen_0 \left(\exp \left(\frac{ze\varphi_i}{k_BT} \right) + \exp \left(-\frac{ze\varphi_i}{k_BT} \right) \right). \tag{7}$$

Here z is the valence of ions in the symmetric electrolyte, e the charge of a proton, n_0 the bulk ionic concentration, k_B the Boltzmann constant, and T the temperature. The convective terms
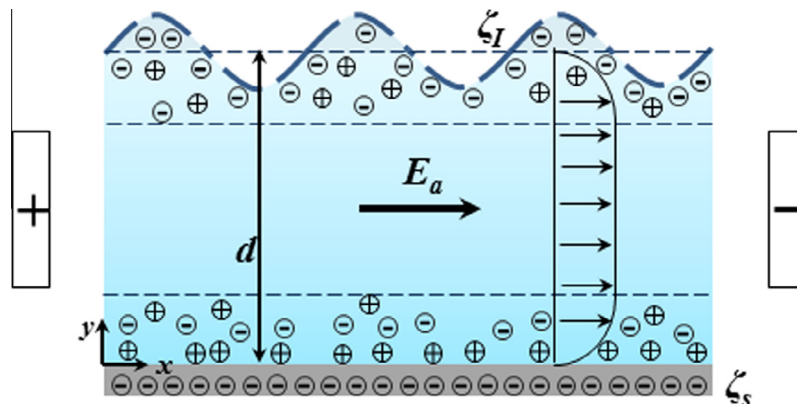

Fig. 1. Schematic diagram of an electrolyte undergoing electro-osmotic flow. An externally applied electro-static field strength 'E_a' is applied across the horizontal cross section of a fluid of thickness 'd'. The zeta potentials at the bottom substrate and free interface are denoted by ζ_S and ζ_I, respectively. The plug-like flow profile of the EOF is denoted by horizontal arrows.