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# Counter-current parallel-plate moving bed heat exchanger: An analytical solution

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ABSTRACT

Counter-flow moving bed heat exchangers provide thermal and economic advantages over their co-current counterparts. Recently, Laplace transforms were used to develop an analytical solution for the set of coupled non-homogeneous equations governing co-current systems. In this work, an extension of the analysis for counter-flow configurations is presented, revealing an important dependency on the capacity ratio. The steady-state energy equations for a plate system are formulated and nondimensionalized, and an analytical solution is presented. Temperature functions, and the associated transcendental equations, for the solids and fluid are presented. The analytical expressions depend on the capacity ratio due to its influence on the location and multiplicity of roots in the Laplace domain solution. Limiting cases are explored and contrasted with expressions in the literature. A graphical analysis further depicts some of the representative behaviors of the solution.



## 1. Introduction

In an accompanying study [1], a novel analytical solution for the steady non-homogeneous convection-conduction equations describing energy transport in co-current parallel-plate moving bed heat exchangers (MBHEs) is presented. These systems are widely used in the energy, chemical and mining industries [2]. Applications range from nickel processing [3] to solar radiation capture [4] and drying of woodchips [5]. Compared with fluidized beds, MBHEs offer low investment cost, energy consumption, and maintenance requirements [6]. In these systems, heat is conveyed between a moving granular bed, consisting of solid particles and an interstitial fluid, and a secondary heating or cooling fluid. Given that flow arrangements can also be counter-current or cross-flow in nature, the co-current analysis [1] must be extended to these other configurations.

In the design of heat exchangers, counter-flow orientations can yield higher log mean temperature differences [7]. Compared with co-current configurations, it is possible for the outlet temperature of the cold material to exceed that of the hot one [8]. However, performance expectations are not intuitive. Under co-current conditions, distance can be used to build equilibrium expectations, since both materials move in the same direction. In counter-flow, equivalent considerations are not as easily developed due to the opposing orientation of flow. This is further complicated in MBHEs by the non-homogeneous convective-conductive nature of the governing equations. Nonetheless, given that increased temperature gradients reduce area requirements and capital investments, an extension of the co-current analytical work [1] is in order. This development will create a baseline for sizing and performance analysis of counter-flow MBHEs.

This work presents a novel analytical solution for heat transfer in counter-current plate MBHEs. This geometry is selected for simplicity with respect to the solids flow behavior [9]. The coupled governing energy equations, with boundary conditions, are presented and nondimensionalized. A Laplace transform methodology is applied to obtain an analytical solution. Limiting conditions are considered for the resulting expressions, and compared with those in the literature. Finally, a graphical analysis explores the consistency of the solutions.

## 2. Model development

### 2.1. System description and assumptions

Consider particulate solids and a heating/cooling fluid moving counter-currently inside the plate heat exchanger shown in Fig. 1. Important system dimensions include: the plate half width $w$, plate height $H$, and plate depth $L$. The solids move at a velocity $u_s$, and enter at a temperature $T_{si}$. The fluid moves with mass flow rate $\dot{m}_f$, and enters at a temperature $t_{fi}$.

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**Nomenclature**

| | |
|---|---|
| $a$ | constant, $=NTU \cdot C$ |
| $A_{hx}$ | area of heat exchange |
| $b$ | constant, $=\sqrt{\frac{Bi}{NTU}}$ |
| $Bi$ | Biot number, $=\frac{U_o \cdot w}{k_s}$ |
| $C$ | capacity ratio, $=\frac{\dot{m}_s \cdot C_{ps}}{\dot{m}_f \cdot C_{pf}}$ |
| $C_{pf}$ | fluid specific heat capacity |
| $C_{ps}$ | solids "effective" specific heat capacity |
| $H$ | plate height |
| $i$ | imaginary number, $=\sqrt{-1}$ |
| $j$ | integer number |
| $k$ | number of multiple roots $s_n$ roots in $\psi$ |
| $k_s$ | solids "effective" thermal conductivity |
| $L$ | plate depth |
| $\dot{m}_f$ | fluid mass flow rate |
| $\dot{m}_s$ | solids mass flow rate |
| $n$ | integer number, positive |
| $NTU$ | Number of Transfer Units, $=\frac{U_o \cdot A_{hx}}{\dot{m}_s \cdot C_{ps}}=\frac{U_o \cdot H}{\rho_s u_s w C_{ps}}$ |
| $s$ | Laplace domain axial variable |
| $s_n$ | simple roots of expansion theorem denominator function $\psi$ |
| $T_s$ | solids temperature function |
| $\overline{T_s}$ | solids average temperature function |
| $T_{si}$ | solids entrance temperature |
| $\overline{T_{so}}$ | solids average outlet temperature |
| $t_f$ | fluid temperature function |
| $t_{fi}$ | fluid entrance temperature |
| $t_{fo}$ | fluid outlet temperature |
| $U_o$ | overall heat transfer coefficient |
| $u_s$ | solids velocity |
| $w$ | plate half width |
| $x$ | axial spatial coordinate |
| $x^*$ | dimensionless axial spatial coordinate, $=\frac{x}{H}$ |
| $y$ | lateral spatial coordinate |
| $y^*$ | dimensionless lateral spatial coordinate, $=\frac{y}{w}$ |

*Greek letters*

| | |
|---|---|
| $\alpha_s$ | solids effective thermal diffusivity |
| $\theta_f$ | dimensionless fluid temperature function, $=\frac{t_f-t_{fi}}{T_{si}-t_{fi}}$ |
| $\theta_{f,Bi\to 0}$ | dimensionless fluid temperature function for a Biot number of zero |
| $\theta_{f,C\to 0}$ | dimensionless fluid temperature function for a capacity ratio of zero |
| $\theta_{f0}$ | dimensionless fluid exit temperature, $x^*=0$ |
| $\theta_{f0,Bi\to 0}$ | dimensionless fluid exit temperature for a Biot number of zero |
| $\theta_{f0,C\to 0}$ | dimensionless fluid exit temperature for a capacity ratio of zero |
| $\tilde{\theta}_f$ | Laplace domain dimensionless fluid temperature function |
| $\theta_s$ | dimensionless solids temperature function, $=\frac{T_s-t_{fi}}{T_{si}-t_{fi}}$ |
| $\theta_{s,Bi\to 0}$ | dimensionless solids temperature function for a Biot number of zero |
| $\theta_{s,C\to 0}$ | dimensionless solids temperature function for a capacity ratio of zero |
| $\tilde{\theta}_s$ | Laplace domain dimensionless solids temperature function |
| $\overline{\theta_s}$ | dimensionless solids average temperature function |
| $\lambda_n$ | $n$th eigenvalue |
| $\mu$ | positive eigenvalue for $C>1$ case |
| $\rho_f$ | fluid density |
| $\rho_s$ | solids "effective" density |
| $\varphi$ | expansion theorem numerator function |
| $\psi$ | expansion theorem denominator function |

The energy model assumptions are the following:

(1) Steady-state conditions.
(2) The moving solids and interstitial fluid are in local thermal equilibrium and behave as a single component continuum [10–14]. The range of validity of this assumption could be studied subsequently by means of, for example, volume averaging techniques [15]. Constraints associated with these conditions could be developed following the work in other porous media studies [16,17]. This description for the solids has been experimentally validated by Sullivan and Sabersky [13], under the necessary presence of a contact resistance at the wall, accounting for variations in the granular structure (see assumption #11).
(3) Thermo-physical properties of the solids (thermal conductivity $k_s$, density $\rho_s$, and specific heat capacity $C_{ps}$) are "effective" and constant. As demonstrated by Quintard and Whitaker [18], under local thermal equilibrium the magnitude of the effective properties is a function of various solids and interstitial fluid properties. The total effective thermal conductivity under this condition is a lumped combination of conductivity properties, tortuosity and fluid dispersion effects. Quantification of these effects in moving bed systems has yet to be explored in detail.
(4) Only lateral ($y$-direction) heat conduction occurs in the solids. Axial conduction ($x$-direction) is negligible relative to convection.
(5) Convection in the solids occurs in the axial $x$-direction only.
(6) Solids move at a constant velocity in the $x$-direction, in agreement with experimental observations for flow over smooth plates [9]. This assumption could be refined in subsequent work, accounting for complex random lateral particle motion. Work of this kind would follow analogously to that of Jackson [19] for fluidized systems.
(7) The temperature profile is symmetric at the midpoint or origin of the solids domain (i.e. $y=0$).
(8) Energy transport in the fluid takes place via convection in the $x$-direction only.
(9) The bulk fluid phase is considered to be well-mixed, and heat exchange between the wall and the bulk is described by a convective heat transfer coefficient. Seider–Tate style correlations can be used to quantify this coefficient, as a function of the flow regime, and equivalent hydraulic and heated diameters [20].
(10) Thermo-physical properties of the fluid are constant (i.e. density $\rho_f$, and specific heat capacity $C_{pf}$).
(11) A contact resistance near the wall accommodates departures from the continuum assumption, as previous experimental and analytical work [10,13,14,21,22] has demonstrated complex mechanisms of energy transfer between a heated surface and adjacent particles in a moving bed. The magnitude of the resistance varies with exchanger geometry, solids properties and operating conditions, and would be obtained experimentally. For an order-of-magnitude estimate, correlations are available in the literature [10,13,14,21,22].