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Constructal design of convective cavities inserted into a cylindrical solid body for cooling

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ABSTRACT

This work applies Constructal design to study numerically the geometry of cavities bathed by a fluid with constant heat transfer coefficient that are intruded into a cylindrical solid body. The objective is to minimize the maximum excess of temperature between the solid body and the ambient by morphing the cavity geometry. Internal heat generating is distributed uniformly into the solid body which has adiabatic conditions on the outer surfaces. The total volume and the volume of the cavities are fixed. The cavities are rectangular with variable aspect ratio. The optimized geometry and performance are reported as functions of the ratio between the volume of the cavities and the total volume, the number of cavities and the dimensionless parameter that accounts for the convective heat transfer, λ. The main results indicate that for fixed number of cavities, ϕ_c, and dimensionless parameter λ, there is an optimal number of cavities, N_o, that minimizes the maximum excess of temperature and this optimal number of cavities in general increases as ϕ_c and λ increases.

1. Introduction

The Constructal law of design and evolution [1] has been invoked to discover the best configurations that facilitate the access of its currents in finite size flow systems. This law of physics accounts for the phenomenon of flow configuration in animate and inanimate systems [2–4]. The large amount of work that has been produced related to Constructal law has been reviewed and documented steadily in the literature [5–8]. Recent example of interesting contribution of this growing field is the study of the evolution of airplanes under the light of Constructal law [9].

Constructal design is the method that is used to apply Constructal law [4]. It has been applied when the geometry of finite size flow systems is the key unknown. Examples of the applicability of the method can be found in the discovery of new designs for electronics, fuel cells, and tree networks for transport of people, goods and information [10–16].

Particular interest has emerged in the search for best shapes to increase the heat transfer in fins and cavities, i.e., inverted or negative fins. In this sense Constructal design has been used successfully to find better architectures in simple T-, and Y-shaped, as well as complex assembly of fins configurations, e.g. T–Y-shaped assembly of fins [17–20]. Best architectures for heat generating pieces in circular and squares fins have also been investigated [21,22].

C-, T-, Y-, double Y-, H-, and T–Y-shaped cavities have also been studied [23–28]. Best shapes of isothermal rectangular cavities inserted into a cylindrical body have been calculated for several numbers of cavities [29]. Additional important results related to the improvement of internal and external shapes of cavities have also been shown recently in the literature [30,31].

Constructal design has also been used to discover best shapes of heat generating systems that are cooled by conductive pathways [32–36] or by convection [37–40]. The goal in these researches was to minimize the maximal temperature, i.e., the hot spots.

This numerical work applies Constructal method to find the best configurations of rectangular cavities inserted into a cylindrical solid body with uniform internal heat generation. The cavities are bathed by a fluid with constant heat transfer coefficient. The outer surfaces of the solid body are adiabatic. The total volume and the volume of cavities are constant, but the aspect ratio of the cavity can vary. The purpose is to minimize the maximal excess

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Nomenclature

a	dimensionless parameter, Eq. (26)
A	area (m^2)
h	heat transfer coefficient ($W\,m^{-2}\,K^{-1}$)
H	height (m)
k	body thermal conductivity ($W\,m^{-1}\,K^{-1}$)
L	length (m)
N	number of cavities
q	heat current (W)
T	temperature (K)
V	volume (m^3)
W	width (m)

Greek symbols

θ	dimensionless temperature
ϕ_c	cavity area fraction
λ	dimensionless group ($hA^{1/2}/k$)

Subscripts

max	maximum
c	cavity
m	single optimization/minimization
mm	double optimization
o	optimal
oo	twice optimized

Superscript

$(\tilde{\ })$	dimensionless variables, Eqs. (6), (10), (11)

of temperature between the solid body and the ambient. The number of cavities, the ratio between the volume of the cavities and the total volume and the dimensionless parameter that accounts for the convective heat transfer in the cavities surface are the problem parameters.

It is worth to mention that the problem addressed in this paper is a fundamental problem, i.e., it is not referent to a specific application. It is known that open cavities are regions formed between adjacent fins. Therefore, if the geometric optimization of the individual fin [17–22] is important, then the geometry of the interstices must also be important [23–31]. In the other side, the dimensionless group $\lambda = hA^{1/2}/k$ (A is the total area and k is the thermal conductivity of the material) emerges when the cavities are bathed by a fluid with constant heat transfer coefficient, h, and temperature, T_∞. The effect of this parameter is also investigated because it depends on the flow regime and the properties of the solid and the fluid [41].

2. Geometry of the cylindrical solid body

Consider the cylindrical solid body in Fig. 1. There is a number N of convective C-shaped cavities intruded into the body. The solid is isotropic with a constant thermal conductivity k. It generates heat uniformly at the volumetric rate q''' (W/m^3). The outer surfaces of the heat generating body are perfectly insulated. The generated heat current ($q'''A$) is removed by convective heat transfer through the cavity walls. The third dimension (W) is sufficiently long in comparison with the radius R of the volume occupied by the body, therefore it is assumed a two-dimensional configuration as shown in Fig. 2.

The objective of the analysis is to determine the optimal geometry that is characterized by the minimum maximal excess of temperature $(T_{\max} - T_\infty)/(q'''A/k)$ for several values of convective fluxes imposed at the cavities surfaces. According to Constructal design [4], this optimization can be subjected to two constraints, namely, the total area constraint,

$$A = \pi R^2 \quad (1)$$

and the total cavity area constraint,

$$A_c = NH_0L_0 \quad (2)$$

where N is the number of cavities inserted into the solid. This equation can be replaced by the fraction area that the cavities occupied by the cavities

$$\phi_c = \frac{A_c}{A} \quad (3)$$

The analysis that delivers the dimensionless global thermal resistance as a function of the geometry consists of solving numerically the heat conduction equation along the solid region,

$$\frac{\partial^2\theta}{\partial\tilde{x}^2} + \frac{\partial^2\theta}{\partial\tilde{y}^2} + 1 = 0 \quad (4)$$

where the dimensionless variables are

$$\theta = \frac{T - T_\infty}{q'''A/k} \quad (5)$$

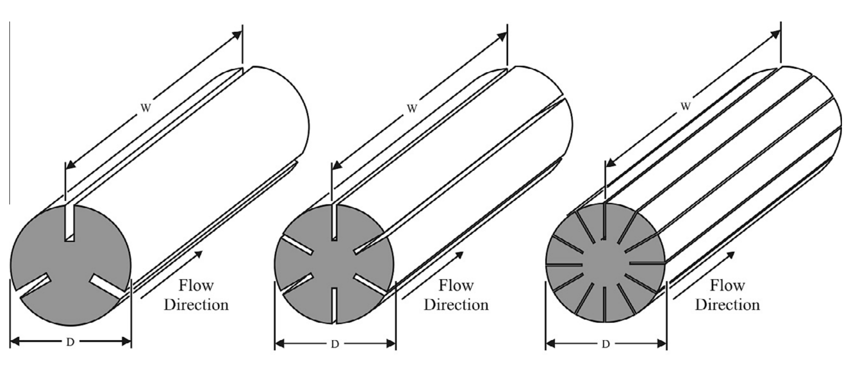

Fig. 1. Examples of convective rectangular cavities inserted to a cylindrical body.