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Local thermal non-equilibrium analysis of the thermoconvective instability in an inclined porous layer

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ABSTRACT

The two-temperature model of local thermal non-equilibrium (LTNE) is employed to investigate the onset of secondary convective flow in a fluid-saturated porous layer inclined to the horizontal and heated from below. The layer is assumed to be bounded by impermeable plane parallel walls with uniform and unequal temperatures. The linear instability of the stationary pure-conduction single-cell basic flow is studied by employing a normal mode decomposition of the disturbances. A Squire-like transformation is adopted to map all the oblique roll modes onto equivalent transverse roll modes. It is shown that the longitudinal rolls are the most unstable modes at the onset of the instability. The neutral stability condition for the longitudinal modes corresponds to that for a horizontal layer, by scaling the Darcy–Rayleigh number with cosine of the inclination angle to the horizontal. This scaling law, coincident with that well-known for the local thermal equilibrium (LTE) regime, implies a monotonic increment in the stability of the basic flow as the inclination to the horizontal increases.

1. Introduction

The study of the onset of thermal instability in a saturated porous layer heated from below is a classical problem in convection heat transfer. This topic deserved a wide space in the literature of the last decades, and it has been reported in many books and review papers such as Nield and Bejan [1], Rees [2], Tyvand [3] and Barletta [4]. A special focus has been made by several authors on the case where a plane porous layer is inclined to the horizontal and bounded by impermeable isothermal walls. A temperature difference between the walls may lead to an unstable stratification that, however, has a nature different from that of a horizontal layer, *viz.* the usual Darcy–Bénard setup [2]. In fact, the inclination to the horizontal results in a basic stationary state where the fluid is not at rest, but circulates along a single cell of infinite width. Strictly speaking, the basic velocity field is parallel, bidirectional, and with a vanishing mass flow rate.

Among the first investigators of the inclined layer instability, we mention Bories and Combarnous [5], Weber [6], Caltagirone and Bories [7]. The main effect of the inclination is that the onset of the thermal instability is with a critical Darcy–Rayleigh number given by $4\pi^2/\cos\phi$, where ϕ is the inclination angle of the layer to the horizontal. In other words, for the onset of the linear instability, a simple scaling law with respect to the Darcy–Bénard problem for a horizontal layer was proved [1]. Further results on the stability of an inclined porous layer were obtained by Storesletten and Tveitereid [8], Karimi-Fard et al. [9], Rees and Bassom [10], and Rees et al. [11]. Karimi-Fard et al. [9] carried out an investigation of oscillatory instability for the case of double-diffusion. Rees and Bassom [10] defined a Squire-like transformation allowing a general study of normal modes with an arbitrary orientation. Storesletten and Tveitereid [8] included in the stability analysis the effect of anisotropy in the porous medium, while Rees et al. [11] extended this analysis by considering an arbitrary orientation of the principal axes of anisotropy.

A recent note by Nield [12] contains new insights into the question of the preferred patterns at the onset of the instability: rolls or polyhedral cells. Nield et al. [13] investigated the influence of the viscous dissipation effect on the onset of instability in an inclined layer. Uniform heat flux boundary conditions, as possible models of the heating from below, were considered in the stability analyses of the inclined porous layer carried out by Barletta and Storesletten [14] and by Rees and Barletta [15].

The aim of this paper is to revisit the topic of instability in an inclined porous layer, by relaxing the assumption that the solid phase and the fluid phase are in local thermal equilibrium (LTE).

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Nomenclature

a	dimensionless wave number
$a_x;\ a_z$	components of the dimensionless wave vector
c	specific heat
$\hat{\mathbf{e}}_x;\ \hat{\mathbf{e}}_y;\ \hat{\mathbf{e}}_z$	unit vectors in the (x,y,z)−directions
$\mathbf{g};\ g$	gravitational acceleration; modulus of $\mathbf{g}$
h	inter-phase heat transfer coefficient
H	dimensionless inter-phase heat transfer parameter, Eq. (2)
k	thermal conductivity
k_m	effective thermal conductivity, $\chi k_f + (1-\chi)k_s$
K	permeability
L	layer thickness
LTE	local thermal equilibrium
LTNE	local thermal non-equilibrium
p	dimensionless pressure disturbance amplitude, Eq. (10)
P	dimensionless pressure disturbances, Eq. (8)
R	Darcy–Rayleigh number, Eq. (2)
$\Re e;\ \Im m$	real part; imaginary part
$S;\ \tilde{S}$	transformed Darcy–Rayleigh numbers, Eqs. (12) and (22)
t	dimensionless time, Eq. (2)
$T_{s,f}$	dimensionless temperatures, Eq. (2)
$\mathbf{u}$	dimensionless velocity, (u,v,w), Eq. (2)
$\mathbf{U}$	dimensionless velocity disturbance, (U,V,W), Eq. (5)
$\mathbf{x}$	dimensionless position vector, (x,y,z), Eq. (2)

Greek symbols

α	thermal diffusivity
α_m	effective thermal diffusivity, $k_m/(\rho c)_f$
β	thermal expansion coefficient
γ	dimensionless parameter, Eq. (2)
ΔT	reference temperature difference
ε	dimensionless perturbation parameter, Eq. (5)
$\eta_{1\ldots4}$	real dimensionless parameters, Eq. (B1)
θ	average dimensionless temperature, Eq. (27)
$\theta_{s,f}$	dimensionless temperature disturbance amplitudes, Eq. (10)
$\Theta_{s,f}$	dimensionless temperature disturbances, Eq. (5)
λ	dimensionless parameter, Eq. (2)
$\Lambda_c(\Phi)$	dimensionless function, Eq. (31)
ν	kinematic viscosity
ρ	density
φ	porosity
ϕ	inclination angle to the horizontal
Φ	transformed angle, Eq. (12)
χ	porosity
ω	complex dimensionless parameter, Eq. (10)

Superscript, subscripts

$^-$	complex conjugate
$\star$	dimensional quantity
b	basic solution
c	critical value
f	fluid phase
s	solid phase
$'$	differentiation with respect to y

Thus, a two-temperature model will be adopted to describe, through a finite inter-phase heat transfer coefficient, the condition of local thermal non-equilibrium (LTNE) [1,16–31]. The study described in this paper is to be considered as a generalisation of the LTNE stability analysis, relative to the Darcy–Bénard problem and hence to a horizontal layer, presented in the paper by Banu and Rees [21].

2. Mathematical model

Let us consider an inclined porous layer saturated by a fluid. We denote as $\phi \in [0^\circ, 90^\circ]$ the inclination angle to the horizontal. The boundary planes, $y^\star = 0, L$, are assumed to be impermeable and isothermal with different temperatures: $T_0 + \Delta T$ is the temperature of the lower boundary, while T_0 is the temperature of the upper boundary, with $\Delta T > 0$. A sketch of the layer is given in Fig. 1.

We assume that the saturated porous medium is isotropic and homogeneous, that the effect of viscous dissipation can be neglected, and that the local thermal non-equilibrium (LTNE) can be described by a two-temperature model [1]. Thus, according to the Oberbeck–Boussinesq approximation, we can write the local mass, momentum and energy balance equations in a dimensionless form as

$$\nabla \cdot \mathbf{u} = 0, \tag{1a}$$

$$\nabla \times \mathbf{u} = \frac{1+\gamma}{\gamma}\, R\, \nabla \times \left[T_f\left(\sin\phi\ \hat{\mathbf{e}}_x + \cos\phi\ \hat{\mathbf{e}}_y\right)\right], \tag{1b}$$

$$\lambda\, \frac{\partial T_s}{\partial t} = \nabla^2 T_s + H\gamma(T_f - T_s), \tag{1c}$$

$$\frac{\partial T_f}{\partial t} + \mathbf{u}\cdot\nabla T_f = \nabla^2 T_f + H(T_s - T_f). \tag{1d}$$

The dimensionless quantities employed in Eqs. (1) are defined as

$$(x,y,z) = (x^\star, y^\star, z^\star)\ \frac{1}{L}, \quad t = t^\star\ \frac{\alpha_f}{L^2},$$
$$\mathbf{u} = (u, v, w) = (u^\star, v^\star, w^\star)\ \frac{L}{\chi\,\alpha_f} = \mathbf{u}^\star\ \frac{L}{\chi\,\alpha_f}, \quad T_{s,f} = \frac{T^\star_{s,f} - T_0}{\Delta T}, \tag{2}$$
$$R = \frac{g\beta\Delta T K L}{\alpha_m\, \nu}, \quad \gamma = \frac{\chi k_f}{(1-\chi)k_s}, \quad \lambda = \frac{\alpha_f}{\alpha_s}, \quad H = \frac{hL^2}{\chi k_f}.$$

Here, the stars denote the dimensional time, coordinates and fields, while the subscripts “s” and “f” denote the solid phase and the fluid phase, respectively. The inter-phase heat transfer coefficient h serves to model the heat exchange between the fluid and the solid phase. We point out that h describes a local volumetric heat transfer

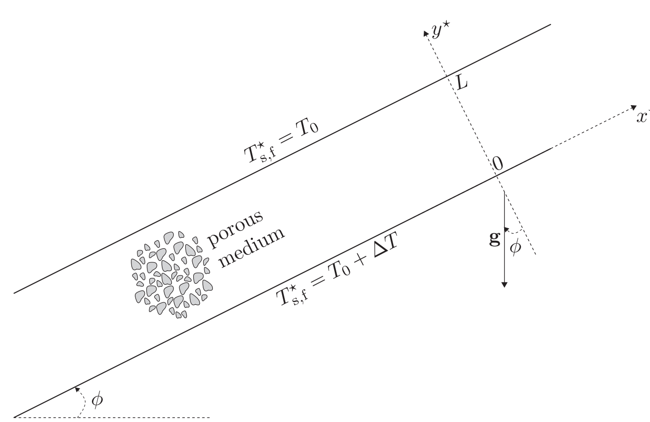

Fig. 1. The fluid saturated porous layer and the thermal boundary conditions.